

КОМПЬЮТЕР ДАСТУРЛАРИ ЁРДАМИДА ФРАКТАЛ ФИГУРАЛАРНИ ТАСВИРЛАШНИ ЎРГАНИШ МЕТОДЛАРИ

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АННОТАЦИЯ

Ҳозирги вақтда "хаотик ночизиқли динамик системалар" математиканинг энг популяр йўналишларидан биридир. Реал ҳодисаларнинг аксарияти чизиқли эмас. Ушбу мақолада биз фрактал фигураларни компьютер дастурлаш орқали тасвирлашни ўрганамиз. Фракталлар чизиқли бўлмаган ҳодисаларга қатъий боғлиқдир. Исаак Нютон даврида ҳар бир реал ҳодиса тартибли ёки турғун эканлиги ҳақидаги илмий қараишлар мавжуд эди. Кейинчалик Пуанкаре [2] кўпгина реал ҳодисалар турғун эмас, яъни улар "тартибсиз" эканлигини исботлади. Биринчи марта фрактал фигуралар компьютерда Бенуа Манделброт томонидан 1980 йилда IBM компаниясининг бир нечта дастурчилари билан кузатилган. Кейинчалик Мандельброт [4] тўплами пайдо бўлди, бу фрактал, Жюлиа [1] тўпламининг энг муҳим хоссалари ва регуляр, тартибсиз ҳодисага боғлиқдир. Ушбу мақолада биз фрактал фигураларни олиш ва уларни табиий ҳодисаларнинг айрим соҳаларида қўллаш учун ишлаб чиқиляётган компьютер дастурини ўрганамиз.

Калит сўзлар: *Исаак Нютон, "хаотик ночизиқли динамик системалар", "тартибсиз".*

МЕТОДЫ ИССЛЕДОВАНИЯ МОДЕЛИРОВАНИЯ ФРАКТАЛЬНЫХ ФИГУР С ПОМОЩЬЮ КОМПЬЮТЕРНЫХ ПРОГРАММ

АННОТАЦИЯ

В настоящее время «хаотические нелинейные динамические системы» являются наиболее популярным разделом математического моделирования. Многие явления реальной жизни нелинейны. В настоящей статье мы исследуем моделирование фрактальных фигур с помощью компьютерного программирования. Фракталы находятся в строгой зависимости с нелинейными явлениями. Во времена Исаака Ньютона существовало научное мнение, что каждое реальное явление закономерно или стабильно. Позднее Пуанкаре [2] заметил, что многие из реальных явлений не являются

регулярными, т. е. «хаотическими». Впервые фрактальные фигуры наблюдал на компьютере Бенуа Мандельброт с несколькими программистами в компании IBM в 1980 году. Позже появляется множество Мандельброта [4], являющееся фрактальным, важнейшим инструментом множеств Жюлиа [1] и строго зависит от нерегулярного явления. В данной работе мы изучаем развивающую компьютерную программу для получения фрактальных фигур и применения их к некоторой области природных явлений.

Ключевые слова: Исаака Ньютона, «хаотические нелинейные динамические системы», «хаотическими».

METHODS FOR STUDYING THE SIMULATIONS OF THE FRACTAL FIGURES WITH COMPUTER PROGRAMS

ABSTRACT

At the present time “chaotic nonlinear dynamical systems” is the most popular branch of the mathematical modeling. Many of real life phenomena are nonlinear. In the present paper we investigate the simulations of fractal figures by computer programming. Fractals are strictly dependence with the nonlinear phenomena. There was scientific view that every real phenomenon is regular or stable at the time of Isaac Newton. Later Poincare [2] observed many of the real phenomena are not regular i.e. they are “chaotic”. At first time fractal figures observed on the computer by Benoit Mandelbrot with several programmers of at the company IBM in 1980. Later appears the set of Mandelbrot [4] which is fractal, the most important tool of the sets of Julia [1] and strictly depends on the irregular phenomenon. In this paper we learn the developing computer program to obtain fractal figures and apply them to some area of natural phenomena.

Keywords: Isaac Newton, “chaotic nonlinear dynamical systems”, “chaotic”.

INTRODUCTION

Let $x_{n+1} = f(x_n, c)$ is the mapping on R to itself.

Definition 1. The set of points $\{x_n \mid x_{n+1} = f(x_n, c)\}$ is called the **orbit** of x_0 for $f(x_n, c)$ mapping.

Definition 2. If the set of points $\{x_n \mid x_{n+1} = f(x_n, c)\}$ if consist only one point then x_0 is called fixed point for $f(x_n, c)$ mapping.

Definition 3. A complex geometric pattern exhibiting self-similarity in that small details of its structure viewed at any scale repeat elements of the overall pattern.

Definition 4. The **filled Julia set** $K(f(x_n, c))$ of a mapping $f(x_n, c)$ is defined as the set of all points $x \in R$, that have bounded orbits with respect to mapping $f(x_n, c)$

$$K(f(x_n, c)) = \{x \mid f^n(x, c) \not\rightarrow \infty \text{ as } n \rightarrow \infty\}$$

Definition 5. The **Julia set** is the common boundary of the filled Julia set

$$J(f(x_n, c)) = \partial(K(f(x_n, c)))$$

Definition 6. The **Mandelbrot set** $M(f(x_n, c))$ for the mapping $f(x_n, c)$ is the set of all points c on the parameter plane (or line), which the orbits of the all critical points are bounded.

Definition 7. If the orbit have following three properties then it is **chaotic**:

- i. Dense periodic points.
- ii. Transitivity.
- iii. Sensitive dependence of initial condition.

1. Algorithms for developing computer programs

First algorithm is for filled Julia set on Euclidean plane for the mapping

$$F : \begin{cases} x_{n+1} = f(x_n, y_n, p) \\ y_{n+1} = g(x_n, y_n, q) \end{cases}$$

where p and q are parameters

Algorithm JS. (For filled Julia set)

```

x=xmin-step
while x< xmax {
y=ymin-step
x=x+step
while y< ymax {
y=y+step
k=0
x1=x
y1=y
while (x1*x1+y1*y1<R) and (k<N) {
k=k+1
xm=x1
x1=f(x1,y1,p)
y1=g(x1,y1,q)
}
if k=N then Print(x,y)
}
}

```

Second algorithm is for Mandelbrot set on Euclidean plane for the mapping

$$F : \begin{cases} x_{n+1} = f(x_n, y_n, p) \\ y_{n+1} = g(x_n, y_n, q) \end{cases}$$

where p and q are parameters.

Algorithm MS. (For Mandelbrot set)

```
x=xmin-step
while x< xmax {
y=ymin-step
x=x+step
while y< ymax {
y=y+step
k=0
x1=x
y1=y
while (x1*x1+y1*y1<R) and (k<N) {
k=k+1
xm=x1
x1=f(x1,y1,p)
y1=g(x1,y1,q)
}
if k=N then Print(p,q)
} }
```

2. Examples

Example 1.

In this example we show filled Julia sets for following mappings on R^2 to itself

$$F : \begin{cases} x_{n+1} = x_n^2 - y_n^2 + p \\ y_{n+1} = 2x_n y_n + q. \end{cases} \quad (1)$$

In our program we chosen $R=6, N=50, x_{min}=-2, x_{max}=2, y_{min}=-2, y_{max}=2,$
 $step=0,0001.$

If $p=q=0$ then filled Julia set is unit circle center on origin Fig 1.

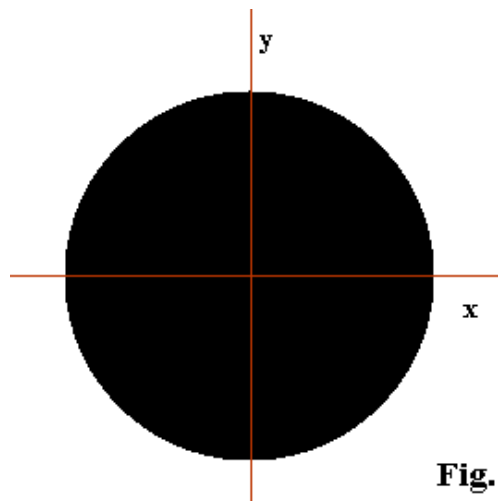
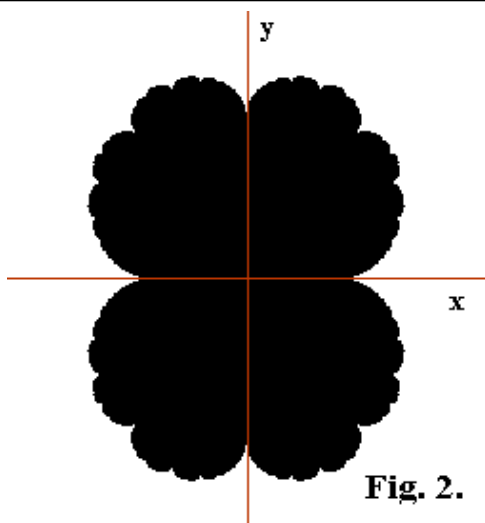
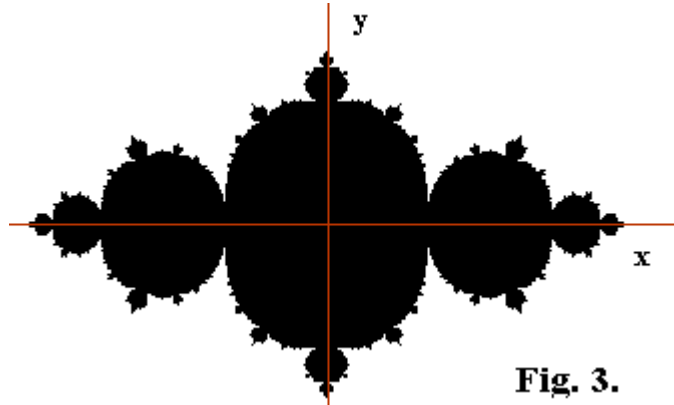


Fig. 1.

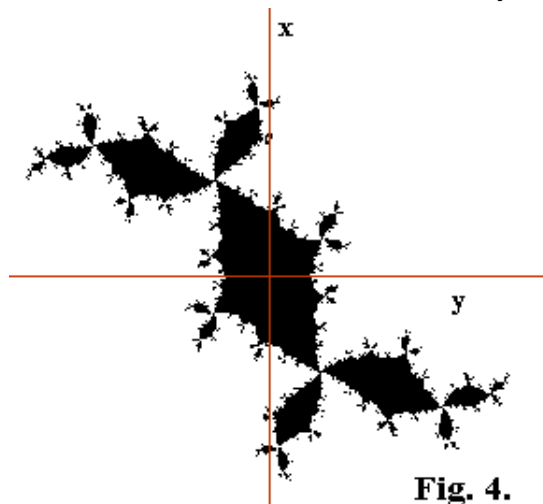
If $p=0,25$ and $q=0$ then filled Julia set is called the “cauliflower” Fig 2 which example of the fractal.



If $p=-0,75$ and $q=0$ then we get Fig 3.



When $p=-0,1$ and $q=0.8$ then our fractal is called Douady's rabbit every where two ears. by name of American mathematics Andrean Douady Fig 4.



When $p= 0,360284$ and $q= 0,100376$ then filled Julia set is in Fig 5.

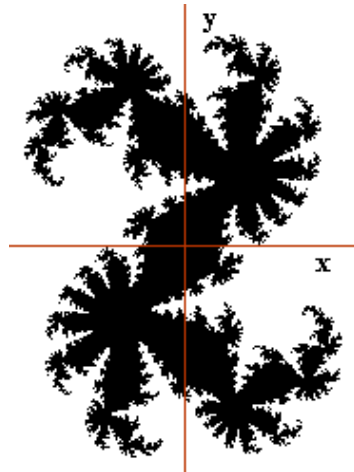


Fig. 5.

The sets of all parameters (p,q) which corresponding filled Julia sets are connected is Mandelbrot set Fig 6.

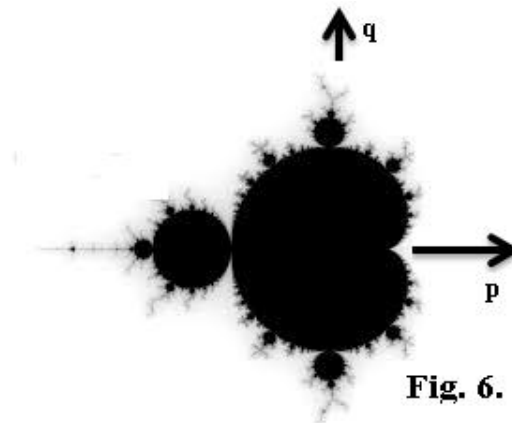


Fig. 6.

Example 2.

In this example we show filled Julia sets for following mappings on R^2 to itself

$$F : \begin{cases} x_{n+1} = y_n^2 + p \\ y_{n+1} = x_n^2 + q. \end{cases} \quad (2)$$

In this case filled Julia sets are regular rectangle for (p,q) in M fig 7.

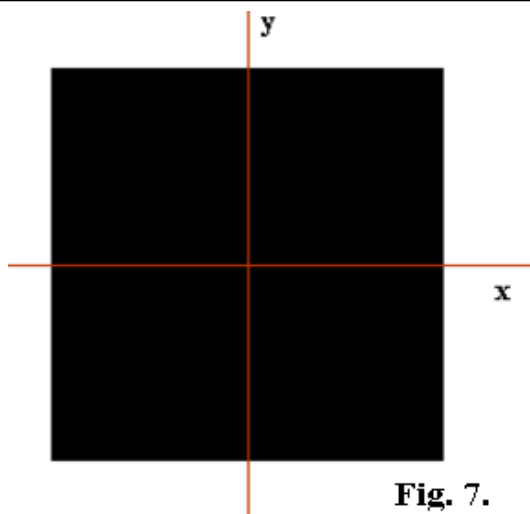


Fig. 7.

And Mandelbrot set for (2) mapping is in fig 8.

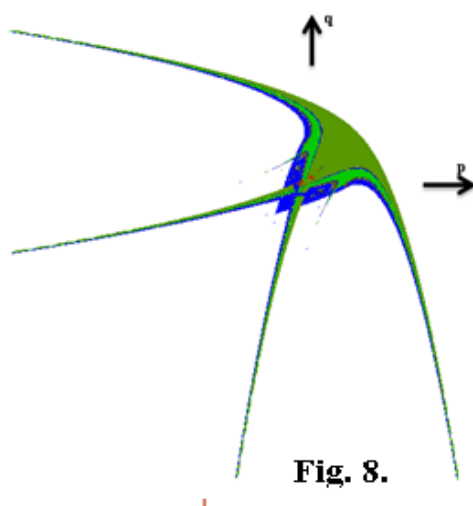


Fig. 8.

Example 3. In this example we consider one dimensional logistic mapping is applicable for epidemic satiation.

$$x_{n+1} = \lambda x_n (1 - x_n) \quad (3)$$

The bifurcation diagram [2] for logistic mapping is in Fig 9.

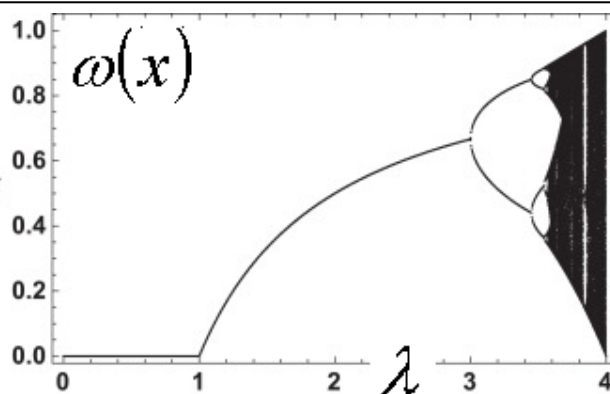


Fig. 9.

We can find proof of these theorem from [1], [3].

Theorem 1. There are exist on boundary orbits of the mapping (1) that they are chaotic.

Theorem 2. If $p=q=-2$ then the orbits of the mapping (2) are chaotic.

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