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OLIMPIADA MASALALARINING YECHIMLARI (UCHBURCHAKLAR)

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ANNOTATSIYA

Mazkur maqolada ko’p uchraydigan uchburchak mavzusiga oid ba’zi olimpiada masalalarning yechimlari keltirilgan. Bular orqali o’quvchi olimpiada masalalarini bir xil usulda emas balki, boshqacha kreativ fikrlash orqali ham yechishi mumkinligini o’rganadi.

Kalit so’zlar: *Uchburchak, tengsizlik, isbot, kosinuslar teoremas, perimetri, bissektrisa, burchak.*

SOLUTIONS OF PROBLEMS IN THE OLYMPIA (TRANGLES)

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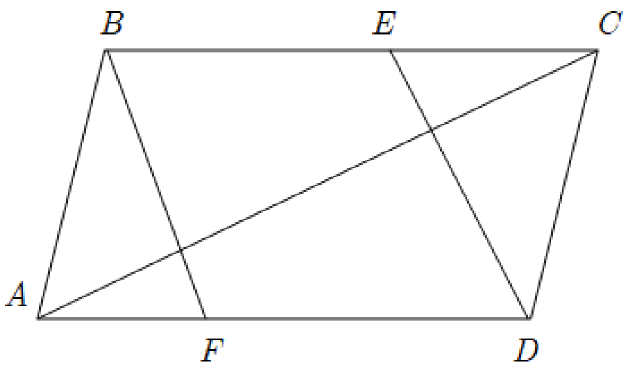
ABSTRACT

This article presents solutions to some of the most common Olympiad problems related to triangles. Through these, the student learns that he can solve Olympiad problems not in the same way, but also through different creative thinking.

Key words: *Triangle, inequality, proof, cosine theorem, perimeter, bisector, angle.*

Ushbu maqola olimpiadada ishtirok etish va g'olib bo'lish istagidagi iqtidorli talabalar uchun yaratilgan. Maqola mustaqil o'rganuvchilar uchun qulay bo'lib, undagi ko'pgina masalalarning yechimlari bilan berilgan.

1. Uchburchakning AA_1va BB_1 bissektrisalarini o'tkazamiz:



N nuqta AB tomonning o'rtasi bo'lsin. Shu nuqtadan AA_1 va BB_1bissektrisalarga NK va NM perpendikulyarlar o'tkazamiz. Bu perpendikulyarlar bissektrisalarini mos ravishda F va E nuqtalar kesib o'tsin.

Birinchidan, AF-umumiy va $< NAF = < KAF$ ekanidan to'g'ri burchakli uchburchaklar tengligining KB(katet-burchak) alomatiga ko'ra, $\Delta ANF = \Delta AKF$ tenglik o'rinli. Bundan BM=BN ekanligi kelib chiqadi. Ikkinchidan, BE-umumiy va $< NBE =< MBE$ ekanidan to'g'ri burchakli uchburchaklar tengligining KB (katet-burchak) alomatiga ko'ra $\Delta BNE = \Delta BME$ tenglik o'rinli. Bundan Bm=Bn ekanligi kelib chiqadi. Shuni isbotlash talab qilingan edi.

Agar AN=Bn ekanini hisobga olsak, Ak=Bm tenglikka ega bo'ladi. Isbot tugadi

2. Birinchi quvur yolg'iz o'zi bo'sh hovuzni x soatda, ikkinchi y soatda, uchinchisi z soatda va uchala quvur birgalikda bo'sh hovuzni, t soatda to'ldirsin deylik. Birinchi kuni hovuz 11 soatda to'lganini hisobga olsak, masala shartidan quyidagi tenglamalar sistemasiga ega bo'lamiz:

$$\begin{cases} \frac{1}{x}*\frac{\frac{2}{3}}{\frac{1}{y}+\frac{1}{z}}+\frac{1}{y}*\frac{\frac{1}{5}}{\frac{1}{x}+\frac{1}{z}}+\frac{1}{z}*\frac{\frac{1}{3}}{\frac{1}{x}+\frac{1}{y}}=1 \\ \frac{\frac{2}{3}}{\frac{1}{y}+\frac{1}{z}}+\frac{\frac{1}{5}}{\frac{1}{x}+\frac{1}{z}}+\frac{\frac{1}{3}}{\frac{1}{x}+\frac{1}{y}}=11 \\ \frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{1}{t} \end{cases}$$

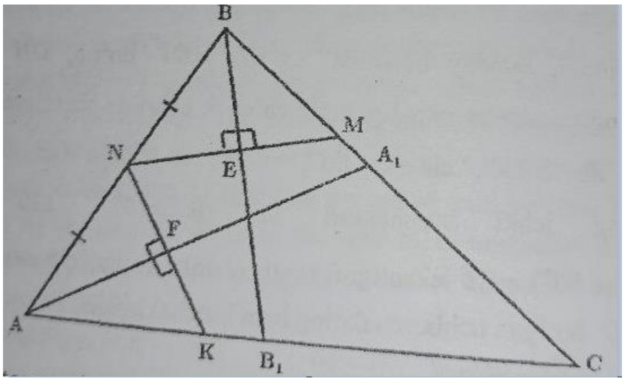
$$\begin{cases} \frac{1}{x}*\frac{2}{3}*\frac{1}{\frac{1}{t}-\frac{1}{x}}+\frac{1}{y}*\frac{1}{5}*\frac{1}{\frac{1}{t}-\frac{1}{y}}+\frac{1}{z}*\frac{1}{3}*\frac{1}{\frac{1}{t}-\frac{1}{z}}=1 \\ \frac{2}{3}*\frac{1}{\frac{1}{t}-\frac{1}{x}}+\frac{1}{5}*\frac{1}{\frac{1}{t}-\frac{1}{y}}+\frac{1}{3}*\frac{1}{\frac{1}{t}-\frac{1}{z}}=11 \end{cases}$$

$$+\begin{cases} \frac{2t}{3(x-t)}+\frac{t}{5(y-t)}+\frac{t}{3(z-t)}=1 \quad /*(-t) \\ \frac{2tx}{3(x-t)}+\frac{ty}{5(y-t)}+\frac{tz}{3(z-t)}=11 \end{cases} \rightarrow \frac{2t}{3}+\frac{t}{5}+\frac{t}{3}=11-t \rightarrow t=5$$

Demak uchala quvur birgalikda bo’sh hovuzni 5 soatda to’ldirar ekan. Hovuz to’lganda soat millari 13^{00} $(8+5=13)$ ni ko’rsatadi.

Javob: Hovuz soat 13^{00} da to’lgan.

3. Masala shartiga mos chizmani chizib olamiz.



Uchburchaklar tengligining TBT (tomo-burchak-tomon) alomatiga ko’ra $\Delta ABF=\Delta CED$ ekanligi kelib chiqadi. U holda qarama-qarshi tomonlari jufti-jufti bilan teng bo’lgan to’rtburchak parallelogramm bo’lishligi alomatidan BEFD to’rtburchak parellelogramm bo’ladi. Bu esa $BF||ED$ ekanini bildiradi.

CAD va BCA burchaklarda Fales teoremasiga ko’ra, AG=GH va GH=HC tengliklari o’rinli. Bu esa AG=GH=HC ekanligini bildiradi. Isbot tugadi.

4. $a,b,x\in R$ uchun |a|+|b| $\geq |a+b|$ va $1\geq|\sin x|$ tengsizliklarga ko’ra, quyidagi yordamchi tengsizlikka ega bo’lamiz:

$|\cos\alpha| + |\cos\beta| = |\cos\alpha| * 1 + |\cos\beta| * 1 \geq |\cos\alpha| * |\sin\beta| + |\cos\beta| * |\sin\alpha| = |\cos\alpha * \sin\beta| + +|\cos\beta * \sin\alpha| \geq |\cos\alpha * \sin\beta + \cos\beta * \sin\alpha| = |\sin(\alpha+\beta)|$

Bundan $\alpha, \beta \in R$ uchun $|\cos\alpha| + |\cos\beta| \geq |\sin(\alpha+\beta)|$ tengsizlikning o'rinli ekani kelib chiqadi. Shunga asosan,

$$|\cos x_1| + |\cos x_2| + |\cos x_3| + |\cos x_4| + |\cos x_5|$$
$$\geq |\sin(x_1+x_2)| + |\sin(x_3+x_4)| + |\cos x_5| =$$
$$= \left|\cos(\frac{\pi}{2} - (x_1+x_2))\right| + \left|\cos\left(\frac{\pi}{2} - (x_3+x_4)\right)\right| + |\cos x_5| \geq$$
$$\geq \left|\sin\left(\pi - (x_1+x_2+x_3+x_4)\right)\right| + |\cos x_5| =$$
$$= \left|\cos\left(\frac{\pi}{2} - (x_1+x_2+x_3+x_4)\right)\right| + +|\cos(-x_5)| \geq$$
$$\geq \left|\sin\left(\frac{\pi}{2} - (x_1+x_2+x_3+x_4+x_5)\right)\right| = \left|\sin\frac{\pi}{2}\right| = 1$$

5. Biror natural sonning kvadrati bo'lgan ikki xonali sonlar 16,25,36,49,64.81 ekanligidan va 11 ga bo'linish qoidasidan foydalanib, quyidagilarni topamiz:

1) b=1, c=6 → $a = 7$ → $\overline{716d}$ → $d = 1$
2) b=2, c=5 → $a = 7$ → $\overline{725d}$ → $d = \emptyset$
3) b=3, c=6 → $a = 9$ → $\overline{936d}$ → d=1
4) b=4, c=9 → $a = 13$, a raqam emas
5) b=6, c=4 → $a = 10,\ a\ raqam\ emas$
6) b=8, c=1 → $a = 9$ → $\overline{981d}$ → $d = 2$

6. $(a-b)^2 \geq 0$ tengsizlikning o'rinli ekanini yaxshi bilamiz. Ikkala tomoniga c^2 ni qo'shamiz va shakl almashtirishlar natijasida quyidagilarga ega bo'lamiz:

$c^2 + (a-b)^2 \geq c^2$ → $c^2 \geq c^2 - (a-b)^2$

→ $c^2 \geq (c-a+b)(c+a-b)$

Xuddi shunga o'xshash $a^2 \geq (a-c+b)(a+c-b)$ *va* $b^2 \geq (b-a+c)(b+a-c)$ tengsizliklar o'rinli ekanini topish mumkin. Hosil bo'lgan tengsizliklarni hadma-had ko'paytirsak, quyidagi muhim tengsizlkika ega bo'lamiz:

$$a^2b^2c^2 \geq (a+b-c)^2(a-b+c)^2(b+c-a)^2$$
$$abc \geq (a+b-c)(a-b+c)(b+c-a)$$

Tengsizlikda tenglik sharti a=b=c bo'lgani bajarildi.

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