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TALABALARGA "MATEMATIK MAYATNIKNING TEBRANISH QONUNI" MAVZUSINI MATEMATIK USULLAR BILAN TUSHUNTIRISH

Nurmatov O.R., Yulchiyev I.I., Axmadjonov M.F., Xidirov D.Sh., Nasirov M.X.
Farg'ona politexnika instituti
ozodnur91@gmail.com

ANNOTATSIYA

Tebranishlar tabiatdagi juda ko'plab moddalarning fizik xususiyatlarini tushuntirib beradi. Shu sabab tebranish qonunlarini o'rganish muhim ahamiyat kasb etadi. Ko'plab abiturientlar va talabalar eng soda tebranuvchi sistemalardan biri bo'lgan matematik mayatnikning tebranish qonunlari natijalarini yodlab olganlar va bu natijalar qanday kelib chiqqani albatta ularni qiziqtiradi. Ammo, bu yechimlarni maktab kursidagi matematik apparatlar bilan topib bo'lmaydi. Maqolada ana shu natijalarni qanday olinishi mumkin qadar matematik usullar bilan keltirib chiqarilgan.

Kalit so'zlar: *Matematik mayatnik, erkin tebranishlar, majburiy tebranishlar, garmonik tebranishlar, angarmonik tebranish, tebranish fazasi, tebranish amplitudasi, tangensial tezlanish, inersia kuchi, elastiklik kuchi, eksponensial funksiya, differensial tenglama.*

EXPLAIN TO STUDENTS BY MATHEMATICAL METHODS THE TOPIC "THE LAW OF OSCILLATIONS OF A MATHEMATICAL PENDULUM"

Nurmatov O.R., Yulchiyev I.I., Axmadjonov M.F., Xidirov D.Sh., Nasirov M.X.
Fergana Polytechnic Institute
ozodnur91@gmail.com

ABSTRACT

Vibrations explain the physical properties of many substances in nature. It is therefore important to study the laws of vibration. Many entrants and students have memorized the results of the laws of vibration of a mathematical pendulum, one of the simplest vibration systems, and are interested in how these results came about. However, these solutions cannot be found with explanations in a school math course. The article shows how these results can be obtained by mathematical methods.

Keywords: *Mathematical pendulum, free oscillations, forced oscillations, harmonic oscillations, enharmonic oscillations, oscillation phase, oscillation amplitude, tangential acceleration, inertia force, elastic force, exponential function, differential equation.*

РАЗЪЯСНИТЬ СТУДЕНТАМ МАТЕМАТИЧЕСКИМИ МЕТОДАМИ ТЕМУ «ЗАКОН КОЛЕБАНИЙ МАТЕМАТИЧЕСКОГО МАЯТНИКА»

Нурматов О.Р., Юлчиев И.И., Ахмаджонов М.Ф., Хидиров Д.Ш., Насиров М.Х.
Ферганский политехнический институт
ozodnur91@gmail.com

АННОТАЦИЯ

Вибрации объясняют физические свойства многих веществ в природе. Поэтому важно изучить законы вибрации. Многие абитуриенты и студенты запомнили результаты законов вибрации математического маятника, одной из простейших систем вибрации, и интересуются, как эти результаты были получены. Однако эти решения не могут быть найдены с объяснениями в школьном курсе математики. В статье показано, как эти результаты могут быть получены математическими методами.

Ключевые слова: *Математический маятник, свободные колебания, вынужденные колебания, гармонические колебания, ангармонические колебания, фаза колебаний, амплитуда колебаний, тангенциальное ускорение, сила инерции, сила упругости, экспоненциальная функция, дифференциальное уравнение.*

KIRISH

Bugungi kunda dars oʻtish jarayonida oʻquvchilarga mavzuni tushuntirishning bir necha usullari mavjud. [3-5] Ushbu maqolada oʻquvchilarga “Matematik mayatnikning tebranish qonuni” mavzusini tushuntirishda, tebranish qonunlarining natijalarini kelib chiqishi va mavzuning tub mohiyatini matematik usullar bilan tushuntirib berilgan.

Bilamizki muvozanat vaziyatiga nisbatan davriy takrorlanib turuvchi harakat turiga ***tebranish*** deyiladi. Muvozanat vaziyatidan chiqarilgan tizimda (tashqi kuchlar ta’sirisiz) ichki kuchlar ta’sirida hosil boʻladigan tebranishlar ***erkin tebranishlar*** deyiladi. Tashqi davriy kuch ta’sirida yuzaga keladigan tebranishlar ***majburiy tebranishlar*** deyiladi. Tebranuvchi sistemaning potensial energiyasi koordinata funksiyasi xisoblanadi va shu sabab uni *U=U(r)=U(x,y,z)* ko‘rinishda yozish mumkin. Agar massa bir o‘lchovli bo‘lsa, potensial energiya ifodasini *U(r)=U(x)* ko‘rinishda yozish mumkin. [1] Bir o‘lchovli kichik tebranishdagi sistemaning potensial energiyasini minimum atrofida qatorga yoyilsa,

$$U(x)=U(0)+\frac{1}{1!}\frac{\partial U(x)}{\partial x}\bigg|_{x=0}x+\frac{1}{2!}\frac{\partial^2 U(x)}{\partial x^2}\bigg|_{x=0}x^2+\frac{1}{3!}\frac{\partial^3 U(x)}{\partial x^3}\bigg|_{x=0}x^3+\ldots \qquad (1)$$

ifoda xosil bo‘ladi. Bunda x -muvozanat xolatdan qancha masofaga chetlashishni bildiradi. (1) qatorning birinchi uchta hadi e’tiborga olinganda keyingi hadlari cheksiz kichik deb olingan holatda yuz beruvchi tebranishlar garmonik tebranishlar deyiladi. Ma’lumki, kuch bu potensial funksiyadan olingan birinchi tartibli xosilaning qarama-qarshi ishoralisiga teng. Shunday ekan yuqoridagi (1) ifodadan birinchi tartibli xosila olamiz;

$$F(x)=-\frac{\partial U}{\partial x}=-\frac{\partial U(0)}{\partial x}-\frac{\partial U(x)}{\partial x}\bigg|_{x=0}-\frac{1}{1!}\frac{\partial^2 U(x)}{\partial x^2}\bigg|_{x=0}x-\frac{1}{2!}\frac{\partial^3 U(x)}{\partial x^3}\bigg|_{x=0}x^2+\ldots=\frac{\partial U(x)}{\partial x}\bigg|_{x=0}-$$
$$-\frac{1}{1!}\frac{\partial^2 U(x)}{\partial x^2}\bigg|_{x=0}x-\frac{1}{2!}\frac{\partial^3 U(x)}{\partial x^3}\bigg|_{x=0}x^2+\ldots=F(0)+\frac{1}{1!}\frac{\partial F(x)}{\partial x}\bigg|_{x=0}x+\frac{1}{2!}\frac{\partial^2 F(x)}{\partial x^2}\bigg|_{x=0}x^2+\ldots\ldots$$

(2)

Yuqoridagi ifodada $\frac{\partial U(0)}{\partial x}=F(0)=0$, chunki o‘zgarmas sonning xosilasi nolga teng.

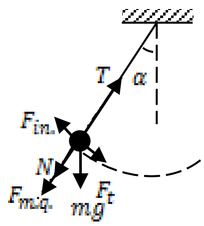
$$F(x)=F(0)+\frac{1}{1!}\frac{\partial F(x)}{\partial x}\bigg|_{x=0}x+\frac{1}{2!}\frac{\partial^2 F(x)}{\partial x^2}\bigg|_{x=0}x^2+\ldots \qquad (3)$$

(3) ifodaning birinchi xadidan tashqari qolganlarini cheksiz kichik xad deb olingandagi tebranishlar chiziqli tebranishlar, aks xolda nochiziqli tebranishlar deb aytiladi. Chiziqli, garmonik tebranishlar ifodasini quidagicha yozib olamiz;

$$F(x)=F(0)+\frac{\partial F(x)}{\partial x}\bigg|_{x=0}x\Rightarrow\left\{F(0)=0,\frac{\partial F(x)}{\partial x}\bigg|_{x=0}=-k\right\}\Rightarrow -kx \qquad (3')$$

Shunday qilib elastiklik kuchi uchun yozilgan biz o‘rganib qolgan kuch ifodasi (3') xosil bo‘ladi.

MUHOKAMA VA NATIJALAR

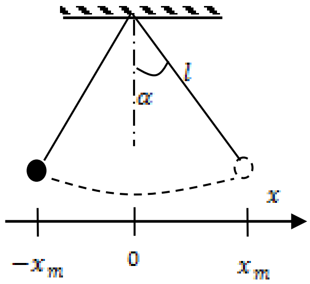


Biz siz bilan yuqorida tebranishlarni asosiy turlarga ajratib oldik. Endi, eng sodda tebranishlardan biri bo‘lgan matematik mayatnikning tebranishi bilan tanishib chiqaylik. Matematik mayatnik og‘irlik kuchi ta’sirida tebranadi va bu tebranishlarni biz kichik sohalarda qarab chiqaylik ya’ni, garmonik tebranishlarini yohud chiziqli tebranishini tekshirishni asosiy masala qilib qo‘yaylik. Asli har qanday tabiat xodisasi juda murakkab. Shu sabab uni o'rganish sodda modellardan boshlanadi. Tebranishlar ham sodda modellardan boshlab o'rganiladi. Ana shu modellardan biri matematik mayatnikdir.

Cho'zilmaydigan, vaznsiz ipga osilgan *m* massali moddiy nuqtalar tizimiga matematik mayatnik deyiladi. Matematik mayatnik og'irlik kuchi ta'sirida tebranadi va matematik mayatnikka ta'sir etuvchi kuchlarning vektorial yig'indisi doimo nolga teng;

$$\vec{N} + \vec{T} + \vec{F}_{im.} + \vec{F}_{m.q.} + \vec{F}_{t.} = 0 \qquad (4)$$

Mayatnik sharchasiga quyidagi kuchlar ta'sir etadi;



- $P=mg$ – sharchaning og'irlik kuchi. U ikki tashkil etuvchiga ajraydi; normal (ip bo'ylab pastga yo'nalgan) va qaytaruvchi (tortuvchi) kuch (ipga tik yo'nalgan)
- $F_{t.}=mgsin\alpha$ – qaytaruvchi (tortuvchi kuch, $\vec{F_t} \perp \vec{T}$)
- $F_{in}=-F_t=-ma_t=-mgsin\alpha$– inersia kuchi, a_t-tangensial tezlanish.
- $T=N+F_{m.q}$ – mayatnik ipining taranglik kuchi.
- $N=mgcos\alpha$ – og'irlik kuchining normal tashkil etuvchisi ($\vec{N} \parallel \vec{F}_{m.q.}$, trarektoriyaga o'tkazilgan urinmaga tik).
- $\vec{F}_{m.q.} = \frac{m\upsilon^2}{l}$ – markazdan qochma kuch.

Matematik mayatnikning harakati uchun Nyutonning 2-qonunini yozaylik;

$$\vec{F} = m\vec{a} \qquad (5)$$

Kinematika kursidan yaxshi ma'lumki tezlanish $a = \frac{d^2x}{dt^2}$ ga teng va matematik mayatnik $F=mgsin\alpha$ kuch ta'sirda tebranishini e'tiborga olib natijalarni (5) ifodaga qo'yib quyidagini olamiz;

$$m\frac{d^2x}{dt^2} = -mg\sin\alpha \qquad (6)$$

Bu ifodani har ikki tarafini *m* ga bo'lamiz, tebranish kichik burchaklar ostida borgani uchun kichik burchaklar sohasida sinα≈α ifoda o'rinli. Misol uchun *α =0.1 rad. ≈sin0.1≈0.099833416* ekanligi ma'lum. (6) tenglikning o'ng tarafini *l* ga ko'paytirib, bo'lamiz (*l*-mayatnik uzunligi). Bunda *αl≈x* ni e'tiborga olamiz (*x*-ihtiyoriy vaqtdagi koordinata). Yuqoridagilarga ko'ra;

$$\frac{d^2x}{dt^2} = -\frac{g}{l}x \qquad (7)$$

(7) ifodada $\omega_0^2 = \frac{g}{l}$ belgilash kiritib olamiz va (7) ifodaning o'ng tomonini chap tarafga olib o'tamiz;

$$\frac{d^2x}{dt^2} + \omega_0^2 x = 0 \qquad (8)$$

Yuqoridagi differensial tenglamani yechish uchun, uning ko'rinishini oldindan taxmin qilib quyidagi eksponensial funksiya ko'rinishida tanlab olamiz;

$$x = e^{kt}, \ \frac{d^2x}{dt^2} = k^2 e^{kt} = k^2 x \qquad (9)$$

(9) ning mos hadlarini (8) ifodaga qo'yib, *k ni* topamiz;

$$k^2 + \omega_0^2 = 0, \ k = \pm i\omega_0 \qquad (10)$$

Bu yerda $i = \sqrt{-1}, \ i^2 = -1$, *i*-mavxum kompleks son. Olingan (10) natijani (9) dagi *k* ning o'rniga qo'ysak, *x* ni ko'rinishini topamiz;

$$x = C_1 e^{-i\omega_0 t} + C_2 e^{-i\omega_0 t} \qquad (11)$$

(11) ifodaga Eyler almashtirishlarini qo'llab, eksponensial funksiya ko'rinishidagi funksiyani trigonometrik ko'rinishga keltirib olamiz;

$$x = C_1 \cos\omega_0 t + iC_1 \sin\omega_0 t + C_2 \cos\omega_0 t - iC_2 \sin\omega_0 t = (C_1 + C_2)\cos\omega_0 t + i(C_1 - C_2)\sin\omega_0 t = A\cos\omega_0 t + B\sin\omega_0 t$$

Biz yuqorida $A = C_1 + C_2, \ B = i(C_1 - C_2)$ deb belgilash kiritdik va x uchun quyidagiga ega bo'ldik;

$$x = A\cos\omega_0 t + B\sin\omega_0 t \qquad (12)$$

x uchun quyidagi chegaraviy shartlarni kiritaylik;

$$x\big|_{t=0} = x_m, \ \frac{dx}{dt}\bigg|_{t=0} = 0 \qquad (13)$$

Bunga ko'ra

$x\big|_{t=0} = A\cos\omega_0 t + B\sin\omega_0 t = A\cos\omega_0 0 + B\sin\omega_0 0 \Rightarrow (bu\ yerda \sin 0 = 0, \cos 0 = 1) \Rightarrow A = x_m$ demak, A=x_m ekan. Ikkinchi shartga asosan

$\frac{dx}{dt}\bigg|_{t=0} = -A\omega_0 \sin\omega_0 t + B\omega_0 \cos\omega_0 t = -A\omega_0 \sin\omega_0 0 + B\omega_0 \cos\omega_0 0 = B\omega_0 = 0$ ma'lumki $\omega_0 \neq 0$, demak, *B=0* ekan. Tebranish qonunining yakuniy ifodasini ham xosil qilib oldik va uning ko'rinishi bunday ekan;

$$x = x_m \cos\omega_0 t \qquad (14)$$

Trigonametriya kursidan ma’lumki *cosx* funksiyaning davri 2π ga teng. U holda $x = x_m \cos\omega_0 t$ funksiyaning davri $T\omega_0 = 2\pi,\ T = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{l}{g}}$ ekanligini ko‘rish unchalik ham qiyin emas. Bundan matematik mayatnikning tebranish davri ham;

$$T = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{l}{g}} \quad (15)$$

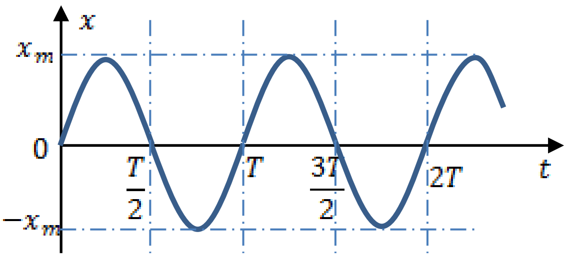
o‘rinli degan xulosa qilish mumkin. [2]

Yakunda matematik mayatnikning tebranish qonunini bunday ko‘rinishga keldi:

$$x = A\cos(\sqrt{\frac{g}{l}}t + \alpha_0) \quad (16)$$

(16) ifodada α_0-tebranishlarning boshlang‘ich ($t=0$) momentdagi fazasi.

Xulosalar bunday, 1) garmonik yoki chiziqli tebranishlar tenglamasi sinuslar yoki kosinuslar qonuniga muvfiq borar ekan, 2) tebranishlar kichik burchak ostida borganda matematik mayatnikning tebranish davri $T = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{l}{g}}$ orqali aniqlanar ekan 3) tebranishlarning siklik chastotasi $\omega_0 = \sqrt{\frac{g}{l}}$ orqali aniqlanar ekan. [1]



(14) tenglamadan olingan 1-tartibli xosila tezlik, 2-tartibli xosila tezlanish ifodasini va 2-tartibli xosilani sharcha massasiga ko‘paytmasi sharchaga ta’sir etuvchi kuchni vaqtga bo‘g‘lanish tenglamasini beradi. Endi, (16) tenglamaga asosan $x=f(t)$ bo‘g‘lanish grafigini ko‘raylik, bunda biz tebranishlarni sinuslar qonuniga asosan borishini tasvirladik ($\alpha_0=0$ holat);

XULOSA

Biz sizlar bilan ko‘rib o‘tgan masalada tebranishlarning eng sodda holatini ko‘rib o‘tik. Bu bilan tebranishlar mavzusini to‘la ochib bera olmadik deyish mumkin, chunki tebranishlar olami juda ham keng va qiziqarli jarayonlarni qamrab oladi. Tanishganimiz garmonik tebranish sohasi edi, agar angarmonik tebranishlar sohasini ham analiz qilsak yanada qiziqarli faktlarga duch kelishimiz mumkin. Buning uchun siz va bizdan harakatdan to‘htamaslik va matematik bilimlarni qo‘llashni soddaroq usullarini topish talab qilinadi.

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