# IKKI O‘LCHOVLI SIMPLEKSDA ANIQLANGAN KVAZI NOVOLTERRA KUBIK STOXASTIK OPERATORINING DINAMIKASI

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## ANNOTATSIYA

*Ushbu maqolada matematikaning zamonaviy tatbiqlaridan biri novolterra kubik stoxastik operatorlarni kvazi sharti ostida ikki o‘lchovli simpleksdagi dinamikasi o‘rganilgan. Shuningdek, kvazi novolterra kubik stoxastik operatorning qo‘zg‘almas nuqtasining yagonaligi haqida teorema isbotlangan.*

***Kalit so‘zlar:*** *kubik operator, kvazi, novolterra, qo‘zg‘almas nuqta, stoxastik.*

## ABSTRACT

*The paper studies the dynamics of non-Volterra cubic stochastic operators in a two dimensional simplex under quasi conditions, one of the modern applications of mathematics. A theorem on the uniqueness of a fixed point of a quasi non-Volterra cubic stochastic operator is also proved.*

***Keywords:*** *cubic operator, quasi, non-Volterra, fixed point, stochastic.*

## АННОТАЦИЯ

*В статье исследуется динамике невольтерровых кубических стохастических операторов в двумерном симплексе при квазиугловиях, одным из современных приложений математики. Также доказана теорема о единственности неподвижной точки квази невольтеррова кубического стохастического оператора.*

***Ключевые слова:*** *кубический оператор, квази, невольтерра, неподвижная точка, стохастический.*

Quyidagi

$$S^{n-1}=\left\{x=(x_1,x_2,...,x_n)\in \mathbb{R}^n : x_i\geq 0, i=\overline{1,n}, \sum_{i=1}^{n} x_i=1\right\}$$

to‘plam $(n-1)$ o‘lchovli *simpleks* deb ataladi [3].

$W:S^{n-1}\to S^{n-1}$ akslantirish,

$$(Wx_l)=x_l'=\sum_{i,j,k=1}^{n} P_{ijk,l}x_ix_jx_k, l=\{1,2,...,n,\} \qquad (1)$$

bu yerda:

$$P_{ijk,l}=P_{jik,l}=P_{kji,l}=P_{kij,l}=P_{jki,l}=P_{ikj,l}\geq 0,\ \sum_{l=1}^{n}P_{ijk,l}=1 \qquad (2)$$

(1), (2) ni *kubik stoxastik operator* deb ataymiz.

Agar $P_{ijk,l}=0,\ \forall l\in\{i,j,k\}$ (3) shart o‘rinli bo‘lsa, (1), (2) operator *novolterra kubik stoxastik operatori* deb ataladi [1], [4].

Agar novolterra operatorining faqat $P_{iii,i}$ va $P_{ijk,l}$, $i\neq j\neq k$ koeffitsiyentlari uchun (3) o‘rinli bo‘lmasa, $W$ operator kvazi novolterra kubik stoxastik operator deb ataladi [4].

Kvazi novolterra kubik stoxastik operatorni $n=3$ da o‘rganamiz:

$$W:\begin{cases}x'=\alpha_1x^3+\beta_1y^3+\gamma_1z^3+3y^2z+3yz^2+2xyz,\\ y'=\alpha_2x^3+\beta_2y^3+\gamma_2z^3+3x^2z+3xz^2+2xyz,\\ z'=\alpha_3x^3+\beta_3y^3+\gamma_3z^3+3x^2y+3xy^2+2xyz,\end{cases} \qquad (4)$$

Bu yerda $\alpha_i,\beta_i,\gamma_i\geq 0,\ i=1,2,3,\ \sum_{i=1}^{3}\alpha_i=\sum_{i=1}^{3}\beta_i=\sum_{i=1}^{3}\gamma_i=1$.

$\alpha_1=\gamma_1=\alpha_2=\beta_2=\beta_3=\gamma_3=0,\ \ \alpha_3=\beta_1=\gamma_2=1$ bo‘lgan holda o‘rganamiz. Ushbu holda operator quyidagi ko‘rinishga keladi:

$$W:\begin{cases}x'=y^3+3y^2z+3yz^2+2xyz\\ y'=z^3+3x^2z+3xz^2+2xyz\\ z'=x^3+3x^2y+3xy^2+2xyz\end{cases} \qquad (5)$$

(5) operatorning qo‘zg‘almas nuqtalarini $W(\lambda)=\lambda,\ \lambda=(x,y,z)$ tenglamani yechish orqali aniqlaymiz. Ya’ni

$$\begin{cases}y^3+3y^2z+3yz^2+2xyz=x\\ z^3+3x^2z+3xz^2+2xyz=y\\ x^3+3x^2y+3xy^2+2xyz=z\end{cases} \qquad (6)$$

$Fix(W)$ orqali $W$ operatorning barcha qo‘zg‘almas nuqtalari to‘plamini belgilaymiz. $Fix(W)=\{\lambda\in S^2:W(\lambda)=\lambda\}$.

Quyidagi belgilashlarni kiritamiz: $\operatorname{int}S^2=\{(x,y,z)\in S^2:xyz>0\}$, $\partial S^2=S^2\setminus\operatorname{int}S^2$ va $C=\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)$.

**Teorema.** (5) operator uchun quyidagilar o‘rinli:

a) $Fix(W)\cap\partial S^2=\varnothing$

b) $Fix(W)\cap \operatorname{int} S^2=\{C\}$.

**Isbot.** a) $(x,y,z)\in\partial S^2$ bo‘lsin. Faraz qilaylik, $x=0$ ( $y=0, z=0$ hollar ham xuddi shunday tekshiriladi) bo‘lsin. U holda (6) sistemaga ko‘ra, tenglamalar sistemasining yechimi $x=y=z=0$ ekanligi kelib chiqadi. Lekin ta’rifga ko‘ra $(0,0,0)\notin S^2$. Bundan ko‘rinadiki,

$$Fix(W)\cap\partial S^2=\varnothing$$

b) $(x,y,z)\in \operatorname{int} S^2$ bo‘lsin. (6) sistemaning birinchi va ikkinchi tenglamalarini ayiramiz:

$$y^3-z^3+3y^2z-3x^2z+3yz^2-3xz^2=x-y$$

yoki

$$(y-z)(y^2+yz+z^2)=(x-y)(1+3z(x+y)+3z^2) \qquad (7)$$

Xuddi shunday (6) sistemaning birinchi va uchinchi tenglamalarini ham ayiramiz:

$$y^3-x^3+3y^2z-3x^2y+3yz^2-3xy^2=x-z,$$

$$(y-x)(y^2+xy+x^2)=(x-z)(1+3y^2+3y(x+z)) \qquad (8)$$

Ikkinchi va uchinchi tenglamalarning ham ayirmasini sodda holga keltiramiz:

$$(z-x)(z^2+zx+x^2)=(y-z)(1+3x(y+z)+3x^2) \qquad (9)$$

$\forall(x,y,z)\in \operatorname{int} S^2$ uchun,

$$y^2+yz+z^2>0,\ \ y^2+xy+x^2>0,\ \ z^2+zx+x^2>0,$$

$$1+3z(x+y)+3z^2>0,\ \ 1+3y^2+3y(x+z)>0,\ \ 1+3x(y+z)+3x^2>0.$$

Faraz qilaylik $z\ge x (z\le x)$ bo‘lsin. (7), (8) va (9) tenglamalardan $x\ge y (x\le y)$ va $y\ge z (y\le z)$ ekanligi kelib chiqadi. Bundan esa $z\ge x\ge y\ge z\ (z\le x\le y\le z)$ (10) munosabatga kelamiz. Shunday qilib, (6) tenglamalar sistemasi yagona $x=y=z=\frac{1}{3}$ yechimga ega. Demak, $C=\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)$ nuqta (5) operatorning yagona qo‘zg‘almas nuqtasi bo‘ladi.

Teorema isbotlandi.

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