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IKKI O'ZGARUVCHILI FUNKSIYANING EKSTREMUMIDAN FOYDALANIB, TEKISLIKDAGI IKKITA FIGURA ORASIDAGI MASOFANI TOPISH

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ANNOTATSIYA

Mazkur maqolada tekislikdagi ikkita figura orasidagi masofani topish masalasi qo'yilgan. Bunda dastlab, ikki o'zgaruvchili funksiyaning xususiy hosilalari, ekstremumlari hamda eng katta va eng kichik qiymatlar haqida tushunchalar keltirilgan. Maqolada metrika tushunchasi, metrik fazo va unga doir misollar ham batafsil yoritilgan. Maqola so'ngida, metrik fazodagi ikkita to'plam orasidagi masofani ikki o'zgaruvchili funksiyaning xususiy hosilalari yordamida topish masalasi ko'rilgan.

Kalit so'zlar: *xususiy orttirma, ekstremum nuqtalar, metrika, metrik fazo, cheklanish, to'plam va masofa.*

НАЙТИ РАССТОЯНИЕ МЕЖДУ ДВУМЯ ФИГУРАМИ НА ПЛОСКОСТИ ПО ЭКСТРЕМУМУ ФУНКЦИИ ДВУХ ПЕРЕМЕННЫХ

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АННОТАЦИЯ

В данной статье рассматривается задача нахождения расстояния между двумя фигурами на плоскости. Сначала представлены понятия специальных производных, экстремумов, наибольшего и наименьшего значений функций двух переменных. Понятие метрики, метрического пространства и связанные с ними примеры подробно рассматриваются в статье. В конце статьи была рассмотрена задача нахождения расстояния между двумя множествами в метрическом пространстве с помощью специальных производных функции двух переменных.

Ключевые слова: *частным приращением, точки экстремума, метрика, метрическое пространство, предел, множество и расстояние*

FIND THE DISTANCE BETWEEN TWO FIGURES IN A PLANE USING THE EXTREMUM OF A FUNCTION OF TWO VARIABLES

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ANNOTATION

This article deals with the problem of finding the distance between two figures on a plane. At first, concepts of special derivatives, extrema, and the largest and smallest values of two-variable functions are presented. The concept of metric, metric space and related examples are covered in detail in the article. At the end of the article, the problem of finding the distance between two sets in a metric space using special derivatives of a two-variable function was considered.

Key words: *private increment, extremum points, metric, metric space, constraint, set and distance.*

1. Ikki o‘zgaruvchili funksiyaning birinchi va ikkinchi tartibli xususiy hosilalari

Ikkita o‘zgaruvchili $z = f(x,y)$ funksiya berilgan bo‘lsin. x o‘zgaruchiga Δx orttirma bersak, u holda z funksiya (x_0, y_0) nuqtada x ga nisbatan *xususiy orttirma* deb ataluvchi $\Delta_x z = f(x_0 + \Delta x, y_0) - f(a, y_0)$ orttirmaga ega bo‘ladi. Agar $\lim_{\Delta x \to 0} \frac{\Delta_x z}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x_0+\Delta x, y_0) - f(x_0, y_0)}{\Delta x}$ limit mavjud bo‘lsa, u holda bu limit z funksiyadan (x_0, y_0) nuqtada x bo‘yicha olingan xususiy hosila deyiladi va $\frac{\partial z}{\partial x}$, z'_x yoki $f'_x(x_0, y_0)$ larning biri kabi belgilanadi. Demak, ta’rif bo‘yicha

$$z'_x = \lim_{\Delta x \to 0} \frac{\Delta_x y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x,\ y_0) - f(x_0,\ y_0)}{\Delta x}$$

bo‘lar ekan.

Xuddi shunga o‘xshahs, z funksiyadan (x_0, y_0) nuqtada y bo‘yicha olingan z'_y xususiy hosila

$$z'_y = \lim_{\Delta y \to 0} \frac{\Delta_y z}{\Delta y} = \lim_{\Delta y \to 0} \frac{f(x_0,\ y_0 + \Delta y) - f(x_0,\ y_0)}{\Delta y}$$

kabi topiladi, bu yerda $\Delta_y z = f(x_0,\ y_0 + \Delta y) - f(x_0,\ y_0)$ ifoda $z = f(x,y)$ funksiyaning (x_0, y_0) nuqtada y ning Δy orttirmasiga mos erishgan xususiy orttirmasi.

Xususiy hosilalar dastlabki x, y argumentlarning funksiyalari bo‘lgani uchun bu xususiy hosilalardan yana xususiy hosilalar olinishi mumkin. Ikkinchi marta olingan xususiy hosilalar berilgan funksiyaning *2-tartibli xususiy hosilalari* deyiladi. Bunda turli o‘zgaruvchilarga nisbatan ketma-ket olingan hosilalar *2-tartibli aralash xususiy hosila* deyiladi.

Shunday qilib,

$z'_x = f'_x(x,y), z'_y = f'_y(x,y)$ hosilalar 1-tartibli xususiy hosilalar bo‘lib, ular x va y o‘zgaruvchilarga bo‘g‘liq funksiyalardir.

$z''_{xx} = f''_{xx}(x,y)$, $z''_{xy} = f''_{xy}(x,y)$, $z''_{yx} = f''_{yx}(x,y)$, $z''_{yy} = f''_{yy}(x,y)$ hosilalar 2-tartibli xususiy hosilalar bo‘lib, bu yerda $z''_{xy} = f''_{xy}(x,y)$ va $z''_{yx} = f''_{yx}(x,y)$ hosilalar aralash xususiy hosilalardir.

1.1. Ikki o‘zgaruvchili funksiyaning ekstremum nuqtalari. $z = f(x,y)$ funksiya berilgan bo‘lib, (x_0,y_0) nuqta $f(x,y)$ funksiya aniqlanish sohasining qandaydir ichki nuqtasi bo‘lsin.

1-ta’rif. Agar (x_0,y_0) nuqtaning shunday

$U_\varepsilon(x_0,y_0) = \{(x,y)\colon\ (x-x_0)^2 + (y-y_0)^2 < \varepsilon\}$

ochiq atrofi topilib, bu atrofning (x_0,y_0) dan farqli ixtiyoriy (x,y) nuqtalari uchun $f(x,y) < f(x_0,y_0)$ tengsizlik bajarilaversa, u holda (x_0,y_0) nuqta $z = f(x,y)$ funksiyaning (*lokal*) *maksimum* nuqtasi deyiladi.

2-ta’rif. Agar (x_0,y_0) nuqtaning shunday

$U_\varepsilon(x_0,y_0) = \{(x,y)\colon\ (x-x_0)^2 + (y-y_0)^2 < \varepsilon\}$

ochiq atrofi topilib, bu atrofning (x_0,y_0) dan farqli ixtiyoriy (x,y) nuqtalari uchun $f(x,y) > f(x_0,y_0)$ tengsizlik bajarilaversa, u holda (x_0,y_0) nuqta $z = f(x,y)$ funksiyaning (*lokal*) *minimum* nuqtasi deyiladi.

Funksiyaning maksimum va minimum nuqtalari uning *ekstremum nuqtalari* deyiladi.

1.2. Ikki o‘zgaruvchili funksiya ekstremum nuqtasi bo‘lishning zaruriy sharti. Agar (x_0,y_0) nuqta $z = f(x,y)$ funksiyaning ekstremumi nuqtasi bo‘lsa, u holda

$$\begin{cases} f'_x(x_0,y_0) = 0, \\ f'_y(x_0,y_0) = 0 \end{cases}$$ **(zaruriy shart)**

bo‘ladi.

Teskarisi o‘rinli emas, ya’ni $f_x'(x_0,y_0)=0$ va $f_y'(x_0,y_0)=0$ tengliklardan (x_0,y_0) nuqtaning ekstremum nuqta ekanligi kelib chiqmaydi.

3-ta’rif. Agar (x_0,y_0) nuqta:

1) berilgan $z=f(x,y)$ funksiya aniqlanish sohasining ichki nuqtasi;

2) 1-tartibli xususiy hosilalarning noli, ya’ni $\begin{cases} f_x'(x_0,y_0)=0, \\ f_y'(x_0,y_0)=0 \end{cases}$

bo‘lsa, u holda (x_0,y_0) nuqta $z=f(x,y)$ funksiyaning *statsionar nuqtasi* deyiladi.

4-ta’rif. Agar $z=f(x,y)$ funksiya uchun $f_x'(x_0,y_0)=0$, $f_y'(x_0,y_0)=0$ bo‘lsa, yoki bu xususiy hosilalarning hech bo‘lmaganda bittasi mavjud bo‘lmasa, u holda (x_0,y_0) nuqta $z=f(x,y)$ funksiyaning *kritik nuqtasi* deyiladi.

1.3. Ikki o‘zgaruvchili funksiya ekstremum nuqtasi bo‘lishning yetarli sharti. $z=f(x,y)$ funksiya va uning biror $M(x_0,y_0)$ statsionar nuqtasi berilgan bo‘lsin, u holda $f_x'(x_0,y_0)=0$ va $f_y'(x_0,y_0)=0$ tengliklar bajariladi.

$A=f_{xx}''(M)$, $B=f_{xy}''(M)$, $C=f_{yy}''(M)$, $\Delta=AC-B$ belgilashlarni kiritaylik.

Yetarli shart. Agar:

$\Delta>0$ va $A<0$ bo‘lsa, u holda M nuqta $z=f(x,y)$ funksiyaning maksimum nuqtasi bo‘ladi;

$\Delta>0$ va $A>0$ bo‘lsa, u holda M nuqta $z=f(x,y)$ funksiyaning minimum nuqtasi bo‘ladi.

Agarda $\Delta<0$ bo‘lsa, unda $z=f(x,y)$ funksiya $M(x_0,y_0)$ nuqtada ekstremumga ega emas deyiladi.

1.4. Ikki o‘zgaruvchili funksiyaning eng katta va eng kichik qiymatlari. $z=f(x,y)$ funksiyaning qandaydir to‘plamdagi eng katta va eng kichik qiymatlarini topish uchun quyidagi amallarni bajarish kerak:

a) to‘plamning barcha ichki kritik nuqtalarini topiladi;

b) funksiya aniqlanish sohasining chegarasida yotuvchi va ikkala xususiy hosilalarni ham nolga aylantiruvchi barcha nuqtalar olinadi;

c) hosil qilingan bu nuqtalar to‘plamidan funksiya eng katta va eng kichik qiymatlarga erishadigan nuqtalar ajratiladi.

2. Metrika tushunchasi

5-ta'rif. X bo'sh bo'lmagan to'plam bo'lsin. Ushbu

$(M1)$ Nomanfiylik:

$\forall x, y \in X: \rho(x, y) \geq 0;$

$(M2)$ Ayniylik aksiomasi:

$\forall x, y \in X: \rho(x, y) = 0 \Leftrightarrow x = y;$

$(M3)$ Simmetriya aksiomasi:

$\forall x, y \in X: \rho(x, y) = \rho(y, x);$

$(M4)$ Uchburchak aksiomasi:

$\forall x, y, z \in X: \rho(x, y) + \rho(y, z) \geq \rho(x, z);$

shartlarni qanoatlantiruvchi $\rho: X \times X \to \mathbb{R}$ funksiya X dagi *metrika* deyiladi. (X, ρ) juftlik *metrik fazo* deyiladi. Qisqalik uchun ko'pincha metrik fazoni bitta X harfi orqali belgilashadi. Metrik fazoning elementlari uning nuqtalari deb yuritiladi.

1-misol. $R = (-\infty, +\infty)$ haqiqiy sonlar to'plami va

$\rho(x, y) = |y - x|$

tenglik bilan aniqlangan $\rho: R \times R \to R$ funksiyani qaraylik.

$\rho(x, y)$ funksiya R da metrika bo'ladi. (R, ρ) metrik fazo $\mathbb{R}$ orqali belgilanadi.

Agar (X, ρ) metrik fazo va $Y \subset X$ bo'lsa, u holda $(Y, \rho|_{Y \times Y})$ juftlik ham X *metrik fazoning qismfazosi* deb ataluvchi metrik fazo bo'ladi. Bu yerda $\rho|_{Y \times Y}$ orqali ρ funksiyaning $Y \times Y \subset X \times X$ qismto'plamga *cheklanishi* belgilangan.

2-misol. $N \times N$ ko'paytmada aniqlangan

$d(m, n) = |m - n|$

funksiya 1-misolda aniqlangan $\rho: R \times R \to R$ funksiyaning cheklaninishi, ya`ni

$d = \rho|_{N \times N}$

funksiya $N = \{1, 2, \dots, n, \dots\} \subset \mathbb{R}$ to'plamda metrika bo'ladi.

(X, ρ) metrik fazoning A qismto'plamini qaraylik. Agar ushbu aniq yuqori chegara

$\sup\{\rho(x, y): x, y \in A\}$

chekli bo'lsa, u holda bu chegara A to'plamning diametri deyiladi va $\operatorname{diam} A$ kabi belgilanadi:

$\operatorname{diam} A = sup\{\rho(x, y): x, y \in A\}.$

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Quyidagi misolda tekislikdagi Evklid metrikasining uchburchak aksiomasini qanoatlantirishi Koshi-Bunyakovskiy tengsizligini qo'llamasdan isbotlandi.

3-misol. Haqiqiy sonlar to'plamining R^2 Dekart ko'paytmasi va

$$\rho(x,y) = \sqrt{(y_1 - x_1)^2 + (y_2 - x_2)^2}$$

tenglik bilan aniqlangan $\rho: R^2 \times R^2 \to R$ funksiyani qaraylik, bu yerda $x = (x_1, x_2), y = (y_1, y_2) \in R^2$. Ravshanki,

$(M1)\ \forall x, y \in X: \quad \rho(x,y) \geq 0$;

$(M2)\ \forall x, y \in X: \quad \rho(x,y) = 0 \Leftrightarrow x = y$;

$(M3)\ \forall x, y \in X: \quad \rho(x,y) = \rho(y,x)$

bo'ladi, ya`ni nomanfiylik, ayniylik, simmetriklik shartlari bajariladi.

Uchburchak aksiomasini, ya`ni

$(M4)\ \forall x, y, z \in X: \rho(x,y) + \rho(y,z) \geq \rho(x,z)$;

tengsizlikni o'rnatish qoldi. Ushbuga egamiz

$$\rho(x,z) = \sqrt{(z_1 - x_1)^2 + (z_2 - x_2)^2} \overset{\boxed{1}}{=} \sqrt{|z_1 - x_1|^2 + |z_2 - x_2|^2} \overset{\boxed{2}}{=}$$
$$= \sqrt{|z_1 - y_1 + y_1 - x_1|^2 + |z_2 - y_2 + y_2 - x_2|^2} \overset{\boxed{3}}{\leq}$$
$$\leq \sqrt{(|z_1 - y_1| + |y_1 - x_1|)^2 + (|z_2 - y_2| + |y_2 - x_2|)^2} \overset{\boxed{4}}{\leq}$$
$$\leq \sqrt{(z_1 - y_1)^2 + (z_2 - y_2)^2} + \sqrt{(y_1 - x_1)^2 + (y_2 - x_2)^2} =$$
$$= \rho(x,y) + \rho(y,z).$$

Barcha $a \in \mathbb{R}$ uchun o'rinli bo'lgan $|a|^2 = a^2$ tenglikdan $\boxed{1}$ tenglik kelib chiqadi.

$\boxed{2}$ tenglik bajarilishi ko'rinib turibdi.

$\boxed{3}$ tengsizlik absolyut qiymatning

$$|a + b| \leq |a| + |b|$$

xossasidan kelib chiqadi, $a, b \in \mathbb{R}$.

$\boxed{4}$ tengsizlikni isbotlash uchun uning ikkala tarafini kvadratga oshiramiz

$$(z_1 - y_1)^2 + 2|z_1 - y_1||y_1 - x_1| + (y_1 - x_1)^2 + (z_2 - y_2)^2 +$$
$$+2|z_2 - y_2||y_2 - x_2| + (y_2 - x_2)^2 \leq$$
$$\leq (z_1 - y_1)^2 + (z_2 - y_2)^2 +$$
$$+2\sqrt{((z_1 - y_1)^2 + (z_2 - y_2)^2)((y_1 - x_1)^2 + (y_2 - x_2)^2)} +$$
$$+(y_1 - x_1)^2 + (y_2 - x_2)^2.$$

Ixchamlashtirgach

$$|z_1 - y_1||y_1 - x_1| + |z_2 - y_2||y_2 - x_2| \leq$$
$$\leq \sqrt{((z_1 - y_1)^2 + (z_2 - y_2)^2)((y_1 - x_1)^2 + (y_2 - x_2)^2)}$$

Hosil bo‘lgan tengsizlikning ikkala tarafini kvadratga oshirib, navbati bilan quyidagi tengkuchli tengsizliklarga ega bo‘lamiz:

$$(z_1 - y_1)^2(y_1 - x_1)^2 + 2|z_1 - y_1||y_1 - x_1||z_2 - y_2||y_2 - x_2| +$$
$$+(z_2 - y_2)^2(y_2 - x_2)^2 \leq$$
$$\leq (z_1 - y_1)^2(y_1 - x_1)^2 + (z_1 - y_1)^2(y_2 - x_2)^2 + (z_2 - y_2)^2(y_1 - x_1)^2 +$$
$$+(z_2 - y_2)^2(y_2 - x_2)^2,$$

$$2|z_1 - y_1||y_1 - x_1||z_2 - y_2||y_2 - x_2| \leq$$
$$\leq (z_1 - y_1)^2(y_2 - x_2)^2 + (z_2 - y_2)^2(y_1 - x_1)^2,$$

$$(|z_1 - y_1||y_2 - x_2|)^2 - 2(|z_1 - y_1||y_2 - x_2|)(|y_1 - x_1||z_2 - y_2|) +$$
$$+(|y_1 - x_1||z_2 - y_2|)^2 \geq 0,$$

$$(|z_1 - y_1||y_2 - x_2| - |y_1 - x_1||z_2 - y_2|)^2 \geq 0.$$

Oxirgi tengsizlik barcha $x = (x_1, x_2), y = (y_1, y_2), z = (z_1, z_2) \in \mathbb{R}^2$ nuqtalar uchun o‘rinli ekanligidan $\boxed{4}$ tengsizlikning to‘g‘ri ekanligi kelib chiqadi. Shu bilan $(M4)$ aksioma (uchburchak aksiomasi) isbotlandi.

Shunday qilib, $R^2 \times R^2$ da aniqlangan

$$\rho(x, y) = \sqrt{(y_1 - x_1)^2 + (y_2 - x_2)^2}, \quad x = (x_1, x_2), y = (y_1, y_2) \in \mathbb{R}^2,$$

funksiya R^2 da metrika bo‘ladi. $\mathbb{R}^2 = (R^2; \rho)$ belgilash kiritamiz.

3. Metrik fazodagi ikkita to‘plam orasidagi masofani topishda ikki o‘zgaruvchili funksiyaning xususiy hosilalarining tatbiqi

Metrik fazodagi ikkita M va N to‘plamlar orasidagi *masofa* deb, ushbu nomanfiy songa aytiladi

$$\rho(M, N) = \inf\{\rho(x, y) : x \in M, y \in N\}.$$

Quyidagi misol to‘plamlar orasidagi masofa tushunchasining ahamiyatini oshirishga xizmat qiladi. Bu misolda umumiy o‘rta ta’lim va oliy ta’lim muassassalari matematika kurslarida uchraydigan amaliy masalalardan biri – ikkita egri chiziq orasidagi masofani topish metodikasi keltiriladi.

4-misol. $\mathbb{R}^2$ dagi $x_2 = \frac{1}{x_1}$ giperbolaning ikkita shoxlari orasidagi masofani toping.

Yechish. Ushbu to‘plamlar

$$M = \left\{\left(x, \frac{1}{x}\right) : x > 0\right\}, \qquad N = \left\{\left(x, \frac{1}{x}\right) : x < 0\right\} \subset \mathbb{R}^2$$

giperbolaning shoxlari bo‘ladi.

$x = (x_1, \frac{1}{x_1}) \in M,\ y = (y_1, \frac{1}{y_1}) \in N$ – ixtiyoriy nuqtalar bo‘lsin. Ushbuga egamiz

$$\rho(x, y) = \sqrt{(y_1 - x_1)^2 + \left(\frac{1}{y_1} - \frac{1}{x_1}\right)^2}\,.$$

Ikkita x_1 va y_1 o‘zgaruvchining $\rho(x, y) = \rho(x_1, y_1)$ funksiyasining eng kichik qiymatini topish talab etiladi. Xususiy hosilalarni topamiz:

$$\frac{\partial \rho}{\partial x_1} = \frac{-2(y_1 - x_1) + \frac{2}{x_1^2}\left(\frac{1}{y_1} - \frac{1}{x_1}\right)}{2 \cdot \sqrt{(y_1 - x_1)^2 + \left(\frac{1}{y_1} - \frac{1}{x_1}\right)^2}} = \frac{\frac{-2(y_1 - x_1)x_1^3 y_1 + 2(y_1 - x_1)}{x_1^3 y_1}}{2 \cdot \sqrt{(y_1 - x_1)^2 + \left(\frac{1}{y_1} - \frac{1}{x_1}\right)^2}} = \frac{(y_1 - x_1)(x_1^3 y_1 + 1)}{x_1^3 y_1 \sqrt{(y_1 - x_1)^2 + \left(\frac{1}{y_1} - \frac{1}{x_1}\right)^2}};$$

$$\frac{\partial \rho}{\partial x_1} = \frac{-2(y_1 - x_1) + \frac{2}{y_1^2}\left(\frac{1}{y_1} - \frac{1}{x_1}\right)}{2 \cdot \sqrt{(y_1 - x_1)^2 + \left(\frac{1}{y_1} - \frac{1}{x_1}\right)^2}} = \frac{\frac{-2(y_1 - x_1)x_1 y_1^3 + 2(y_1 - x_1)}{y_1^3 x_1}}{2 \cdot \sqrt{(y_1 - x_1)^2 + \left(\frac{1}{y_1} - \frac{1}{x_1}\right)^2}} = \frac{(y_1 - x_1)(x_1 y_1^3 + 1)}{x_1 y_1^3 \sqrt{(y_1 - x_1)^2 + \left(\frac{1}{y_1} - \frac{1}{x_1}\right)^2}};$$

Ushbu tenglamalar sistemasini yechamiz

$$\begin{cases} \frac{\partial \rho}{\partial x_1} = 0, \\ \frac{\partial \rho}{\partial x_1} = 0. \end{cases}$$

$x_1 - y_1 \neq 0$ ekanligidan

$$\begin{cases} x_1^3 y_1 + 1 = 0, \\ x_1 y_1^3 + 1 = 0. \end{cases}$$

Birinchi tenglamadan ikkinchisini ayirib, hosil bo‘lgan tenglamani y_1 ga nisbatan yechamiz:

$$x_1^3 y_1 - x_1 y_1^3 = 0,$$

$$x_1 y_1 (x_1^2 - y_1^2) = 0,$$

$$x_1 y_1 (x_1 - y_1)(x_1 + y_1) = 0,$$

$y_1 = -x_1$.

Bunda $x_1 \neq 0, y_1 \neq 0, x_1 - y_1 \neq 0$ ekanligi e'tiborga olindi. ${x_1}^3 y_1 + 1 = 0$ tenglamada y_1 ning o'rniga $-x_1$ qo'yib, x_1 ni va unga mos y_1 ni topamiz:

$-{x_1}^4 + 1 = 0,$

$x_1 = -1, \qquad y_1 = 1;$

$x_1 = 1, \qquad y_1 = -1.$

Biroq $(x_1; \frac{1}{x_1}) \in M$ ekanligidan $x_1 > 0$ bo'ladi. Shuning uchun $x_1 = 1$. Bundan $y_1 = -1$. Shunday qilib, stasionar nuqta: $(1, -1)$.

Ikkinchi tartibli xususiy hosilalarni topamiz:

$$\frac{\partial^2 \rho}{\partial {x_1}^2} = \frac{({x_1}^4 y_1 - 4{x_1}^2 + 3x_1 y_1)\left((y_1 - x_1)^2 + \left(\frac{1}{y_1} - \frac{1}{x_1}\right)^2\right) - 2(x_1 - y_1)^2({x_1}^3 y_1 + 1)^2}{{x_1}^5 y_1 \left((y_1 - x_1)^2 + \left(\frac{1}{y_1} - \frac{1}{x_1}\right)^2\right)\sqrt{(y_1 - x_1)^2 + \left(\frac{1}{y_1} - \frac{1}{x_1}\right)^2}},$$

$$\frac{\partial^2 \rho}{\partial x_1 \partial y_1} =$$

$$= \frac{(x_1^7 y_1 - 2{x_1}^6 {y_1}^2 + {x_1}^5 y_1 - {x_1}^4 y_1 - x_1 + {x_1}^3 {y_1}^2 + y_1)\left((y_1 - x_1)^2 + (\frac{1}{y_1} - \frac{1}{x_1})^2\right) - \frac{{x_1}^2}{{y_1}^2}(x_1 - y_1)({x_1}^3 y_1 + 1)({x_1}^4 y_1 + x_1 - {x_1}^3 {y_1}^2 - y_1)}{{x_1}^6 {y_1}^2 \left((y_1 - x_1)^2 + \left(\frac{1}{y_1} - \frac{1}{x_1}\right)^2\right)\sqrt{(y_1 - x_1)^2 + \left(\frac{1}{y_1} - \frac{1}{x_1}\right)^2}},$$

$$\frac{\partial^2 \rho}{\partial {y_1}^2} = \frac{(x_1 {y_1}^4 - 4{y_1}^2 + 3x_1 y_1)\left((y_1 - x_1)^2 + (\frac{1}{y_1} - \frac{1}{x_1})^2\right) - 2(x_1 - y_1)^2(x_1 {y_1}^3 + 1)^2}{x_1 {y_1}^5 \left((y_1 - x_1)^2 + (\frac{1}{y_1} - \frac{1}{x_1})^2\right)\sqrt{(y_1 - x_1)^2 + \left(\frac{1}{y_1} - \frac{1}{x_1}\right)^2}}.$$

Ushbularni topamiz:

$$A = \left.\frac{\partial^2 \rho}{\partial x_1^2}\right|_{\substack{x_1 = 1, \\ y_1 = -1}} = 4\sqrt{2}, \qquad B = \left.\frac{\partial^2 \rho}{\partial x_1 \partial y_1}\right|_{\substack{x_1 = 1, \\ y_1 = -1}} = -\frac{\sqrt{2}}{2},$$

$$C = \left.\frac{\partial^2 \rho}{\partial y_1^2}\right|_{\substack{x_1 = 1, \\ y_1 = -1}} = 3\sqrt{2}, \qquad \Delta = AC - B^2 = 24 - \frac{1}{2} = 23\frac{1}{2}.$$

$\Delta > 0$, $A > 0$ bo'lgani uchun $(1, -1)$ – minimum nuqtasi bo'ladi. Shunday qilib,

$$\rho(M, N) = \inf_{\substack{(x_1, \frac{1}{x_1}) \in M, \\ (y_1, \frac{1}{y_1}) \in N}} \rho\left(\left(x_1, \frac{1}{x_1}\right), \left(y_1, \frac{1}{y_1}\right)\right) = \rho\big((1,1), (-1,-1)\big) = 2\sqrt{2}.$$

REFERENCES

1. А.А. Заитов. Элементы линейной алгебры и аналитической геометрии. Учебное пособие. – Ташкент: «Zuxra baraka biznes.” – 123 с.

2. А. А. Заитов. Элементы дифференциального исчисления. Учебное пособие. – Ташкент: изд-во ТГПУ. – 131 с.
3. A. A. Zaitov, A. Ya. Ishmetov. Matematika 1. O‘quv qo‘llanma. – Toshkent: “Zuxra baraka biznes” – 225 bet.
4. Бозаров, Д. У. (2022). DETERMINANTLAR MAVZUSINI MUSTAQIL OQISHGA DOIR MISOLLAR. *Журнал Физико-математические науки*, *3*(1).
5. Uralovich, B. D. (2022). CHIZIQLI ALGEBRAIK TENGLAMALAR SISTEMALARIGA OID MASALALAR. *Science and innovation*, *1*(A2), 163-171.
6. Uralovich, B. D., Normamatovich, R. B., & O’Gli, A. Z. A. (2021). SONLARDAN ILDIZ CHIQARISH HAQIDA. *Oriental renaissance: Innovative, educational, natural and social sciences*, *1*(4), 1428-1432.
7. Бозаров Д. У. MATRITSALAR MAVZUSINI MUSTAQIL O‘ZLASHTIRISHGA DOIR MISOLLAR //Муғаллим ҳам узликсиз билимлендириў. – 2022. – Т. 3. – №. 3.
8. Bozarov D. U. CHIZIQLI VA KVADRATIK MODELLASHTIRISH MAVZUSINI MUSTAQIL O’RGANISHGA DOIR MISOLLAR // EURASIAN JOURNAL OF MATHEMATICAL THEORY AND COMPUTER SCIENCES. – 2022. – Т. 2. – №. 6. – С. 24-28.
9. Olimovich T. E., Uralovich B. D., Matlubovich M. J. Effective Methods in Teaching Mathematics //International Journal on Orange Technologies. – Т. 3. – №. 3. – С. 88-90.
10. Uralovich B. D., Normamatovich R. B., Kholmatovich K. J. Development Of Mathematics In Different Periods //European Journal of Research Development and Sustainability. – 2021. – Т. 2. – №. 3. – С. 53-54.
11. Maxmudovna G. M., Olimovich T. E., Uralovich B. D. Types and uses of mathematical expressions //ACADEMICIA: An International Multidisciplinary Research Journal. – 2021. – Т. 11. – №. 3. – С. 746-749.
12. https://t.me/zaamath
13. https://in-academy.uz/index.php/EJMTCS/article/view/2606