Oriental Renaissance: Innovative, educational, natural and social sciences
Research BIB / Index Copernicus
(E)ISSN: 2181-1784
4(10), Nov., 2024
www.oriens.uz

FREE PROBLEM POSING COMPETENCIES OF CLASSROOM TEACHERS IN THE CONTEXT OF MATHEMATICAL PROCESS SKILLS

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ABSTRACT

It is very important to examine the problem posing competencies of classroom teachers since the qualities of the problems that teachers use in classes will affect the quality of mathematics teaching. Therefore, the study aimed to examine the free problems posed by classroom teachers in the measurement learning area in terms of mathematical process skills. A qualitative research design was used with seven volunteer classroom teachers with 15-32 years of seniority who teach at the fourth grade level. The analysis of the data obtained with the test consisting of four problem posing situations belonging to the measurement learning area was carried out through descriptive analysis. In terms of mathematical communication skills, it was observed that classroom teachers made mistakes when using mathematical concepts, symbols and expressions in the problems they posed and generally did not use different types of representations other than verbal representation. While making connections between concepts in problems posed in terms of mathematical association, associations between different representations of concepts were mostly not included. It was revealed that the vast majority of the problems posed in terms of mathematical reasoning required reasoning based on analogy.

Key words: *classroom teachers, problem posing, mathematical communication, mathematical association, mathematical reasoning, measurement.*

INTRODUCTION

Problem posing is defined as "the process by which students formulate their personal interpretations of concrete situations based on their mathematical experiences and formulate them as mathematical problems" (Stoyanova & Ellerton, 1996). Problem posing enriches both teaching and learning and contributes to a deeper understanding of mathematical concepts in the teaching process (Tichá & Hošpesová, 2009). Teachers should be able to blend the mathematical concepts to be taught with daily life in problem posing activities to support teaching (Lin, 2004). Problem posing is a critical element of teachers' work, both in terms of posing problems for students and helping students become better problem posers (Cai et al., 2015; Crespo, 2003; Olson & Knott, 2013; Zhang et al., 2023). The problems a teacher poses can shape mathematical learning in their classrooms and "advance mathematical goals in the classroom" (National Council of Teachers of Mathematics [NCTM], 2000). Problem posing activities contribute to the active use of individuals' thinking processes (Cai & Hwang, 2002; Silver & Cai, 1996), improve problem solving skills (Cai & Hwang, 2002; Cankoy & Darbaz, 2010; Siswono, 2010), allow individuals to relate mathematics to daily life (Lin, 2004; Tichá & Hošpesová, 2009), and can be used as an assessment tool to reveal learning and misconceptions (Hošpesová & Tichá, 2015; Kar, 2014; Lin, 2004; Xie & Masingila, 2017) and teachers and pre-service teachers may be inadequate in terms of problem posing skills (Canbazoğlu & Tarım, 2019; Ellerton, 2013; Hošpesová & Tichá, 2015; Işık & Kar, 2012; Kar, 2014; Korkmaz & Gür, 2006; Serin, 2019; Tichá & Hošpesová, 2009; Tekin-Sitrava & Işık, 2018), improving educators' problem posing skills will contribute to a more qualified teaching. In this respect, the quality of the problems that teachers use in the classroom is important in mathematics teaching (Crespo & Sinclair; 2008; Kar, 2014).

Reasoning, association and communication are among the process skills that students should acquire in mathematics teaching (Ministry of National Education [MoNE], 2013, 2015, 2018). Mathematical communication skill is the ability to use mathematical concepts and language effectively and correctly in the process of revealing individuals' mathematical thinking (Kabael & Baran, 2016; NTCM, 2000). As a result of the correct use of mathematical communication, students realize permanent and meaningful learning. Therefore, teachers and students need to master the language of mathematics for effective mathematics teaching (Erdoğan et al., 2023). Considering the contribution of mathematical concepts to in-depth meaningful learning, mathematical communication skill is a skill that each student should acquire. For this skill, students should be able to use pictures, words, graphics, symbols, etc. related to mathematics (MoNE, 2013). Mathematical association is seen as a process and product, and it is the creation of connections between concepts, daily life and different disciplines (Özgen, 2016). Mathematical association is an indispensable element of the process of doing and teaching mathematics (MoNE, 2013). Students should have mathematical association skills in order to acquire mathematical knowledge, use it in different fields, and make sense of mathematical concepts by making connections between them (MoNE, 2013; Yavuz-Mumcu, 2018). Mathematical association is grouped as internal association and association in different fields. While association within itself is divided into two as association between different representations and association between concepts, association in different fields is divided into two as association with real life and association with different disciplines (Bingölbali & Coşkun, 2016; Yavuz-Mumcu, 2018). Mathematical reasoning, another mathematical process skill, contributes significantly to students' problem solving in logical ways by using mathematical language, establishing patterns and relationships (MoNE, 2009). Mathematical reasoning was evaluated in two main categories: creativity- based and analogy-based reasoning. In analogy-based reasoning, the student will be able to solve the new problem based on the solution of a problem he/she has solved before. The student is not expected to make interpretation and inference in solving the problem. In creativity-based reasoning, on the other hand, it is important for the student to make logical inferences and search for solutions, unlike remembering the solution path that will lead to the answer (Lithner, 2008). Jäder et al. (2017) stated that students could not reach a solution because they preferred analogy-based reasoning when solving non-routine problems. It is possible to talk about analogy-based reasoning in solving routine problems and creativity-based reasoning in solving non-routine problems.

Teachers should be able to pose quality problems to students in order to help them acquire mathematical process skills. Therefore, teachers should be able to construct quality problems (Işık et al., 2011; Kar, 2014; Van de Walle et al., 2012, p.34). However, considering that teachers use mathematics textbooks in the classroom and mathematics textbook review studies, it has been observed that there are not enough quality problems that give students the opportunity to use mathematical process skills (Bütüner, 2019; Özer & İncikabı, 2019; Usta & İpek, 2019). In addition, the distribution of problem posing activities in primary school mathematics textbooks used in teaching is insufficient according to the units and limited in terms of diversity (Karabey, 2020). The problems in mathematics textbooks are very few for associating with different disciplines, insufficient for associating with daily life (Özdiner, 2021), simple exercises, and generally use verbal representation (Karakuzu, 2017).

According to the curriculum, teachers should include problem posing activities in mathematics lessons (MoNE, 2018). However, it has been observed that teachers and prospective teachers are insufficient in terms of problem posing skills (Canbazoğlu & Tarım, 2019; Ellerton, 2013; Hošpesová & Tichá, 2015; Işık & Kar, 2012; Kar, 2014; Korkmaz & Gür, 2006; Serin, 2019; Tichá & Hošpesová, 2009;

Tekin-Sitrava & Işık, 2018). Therefore, there is a need to further investigate how competent teachers are in constructing important and valuable mathematical problems based on different problem situations (Cai & Hwang, 2020; Christou et al., 2024). Cai et al. (2015) suggested that one of the most important questions to be answered in future research is whether teachers can construct important and valuable mathematical problems.

In this study, an answer to the question "How are the competencies of classroom teachers in constructing free problems in the field of measurement learning in the context of mathematical process skills?" was sought. The fact that learners and teachers experience problems (Drake, 2013; Divrik & Pilten, 2021; Doğan-Coşkun, 2017; Kamii & Russel, 2012; Şimşek & Boz, 2015) was effective in the preference of constructed problems in the field of measurement learning. In addition, pre-service primary school teachers stated that the problems they constructed in the measurement learning domain had errors and deficiencies in terms of mathematical communication and association. In terms of reasoning, they generally constructed problems that required reasoning based on analogy (Sayın & Orbay, 2023; Sayın & Orbay, 2024a). Considering the mathematical process skills that students should acquire in the mathematics curriculum, the problems in the measurement learning domain were examined. In this direction, the problems constructed by classroom teachers in the measurement learning domain before and after the problem posing instruction given to the teachers were analyzed.

1. How are their competencies in terms of mathematical communication skills?
2. How are their competencies in terms of mathematical association skills?
3. What are their competencies in terms of mathematical reasoning skills? sub research questions were formed.

METHOD

Research Model

In this study, qualitative research method was preferred in order to examine the free problems constructed by classroom teachers in terms of mathematical process skills. The data obtained from the teachers were subjected to descriptive analysis according to the conceptual framework created by the researchers. The research was conducted with volunteer classroom teachers who participated as a purposive sample. Since the research was in the pandemic process, problem posing teaching was carried out through distance education.

Participant Characteristics

The participants of the study consisted of seven volunteer teachers, four of whom were male and three of whom were female, working as classroom teachers at the fourth grade level of primary school. The professional experience of the teachers ranged between 15 and 32 years. The classroom teachers stated that they had not received problem posing instruction before and that they wanted to improve themselves in this regard. It was observed that the classroom teachers were willing to participate in problem posing instruction.

Data Collection Tools

A problem-posing test (PST) consisting of four problem-posing situations was used as a data collection tool. The problem posing situations were created according to the four learning outcomes of the fourth grade measurement learning domain in the 2018 mathematics curriculum. The preference of these acquisitions in the PCT was influenced by the fact that the 2018 primary school mathematics curriculum states that problem posing activities should be included in the measurement learning domain. Two classroom teachers and a mathematics educator were consulted to ensure the content and face validity of the test.

Application

Teachers participated in this research conducted during the pandemic from their homes. Teachers used their own tools and materials in the research conducted through the program known as 'Zoom' as distance education. For reliability, participants answered the SEP with the screen on. After answering the PCT, the participants sent the image of their answers by e-mail. The researcher then physically collected these answer sheets from the teachers. Thus, it was ensured that no changes were made in the answers.

In the first and last week of the study, which lasted for six weeks, two hours each week, the PCT was administered as a pre-test and post-test. After the pre-test was administered, teachers were informed about the problem concept for one class hour in the first week. In the second week, instruction was given on problem posing and problem types. In the first class hour of the third week, problem solving and problem posing learning outcomes in the measurement learning domain of primary school were stated. Teachers were informed about the limitations of the outcomes at the fourth grade level. Information was given about the concepts and units of measurement. In the second lesson, information was given about mathematical communication, association and reasoning skills. Mathematical process skills were demonstrated on sample problems. In the fourth week, the concepts of mathematicality, contextuality, language and expression, instruction and data quality, conformity to the learning outcomes, and solvability were emphasized. The faulty problems presented by the researchers in the teaching were examined and discussed by the teachers. By ensuring that each participant was active in the process, the problems were corrected and made solvable and appropriate for the students' grade level. Information was given about free problem posing from problem posing situations. Examples of free problem posing situations were given. In the fifth week, it was discussed under the chairmanship of the researchers whether there were errors or deficiencies in the problems that the participants set up. As a result of the exchange of ideas, the problems were corrected. In the last week, the participants continued to discuss the problems and correct the mistakes. Teachers corrected each other's problems during the implementation process and created problems that could be used in the classroom.

Data Collection and Analysis

Due to the pandemic, the SEP was sent to the participants via e-mail before and after the implementation. The teachers filled out the printed SEP within one hour while the screen of the 'Zoom' program was open and sent it to the researchers in the same way. In the study, the problems posed by the classroom teachers were analyzed in terms of mathematical process skills. In the evaluation of the problems, the conceptual framework prepared by the researchers according to the relevant literature (Bingölbali & Coşkun, 2016; Lithner, 2008; MEB, 2013, 2015, 2018; Yavuz-Mumcu, 2018) was used. Figure 1 shows the conceptual framework used in the analysis process.

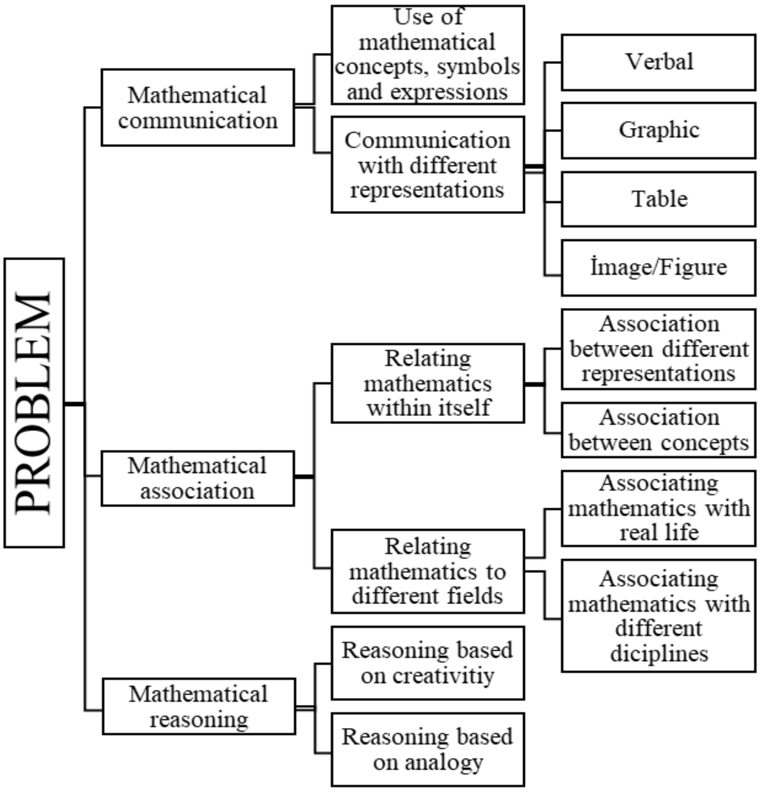

Figure 1. Conceptual framework to be used in the evaluation of problems

The data obtained from the PCT were analyzed descriptively. The frequency of the findings according to mathematical process skills was shown in the table and direct quotations were made to support the data obtained. When the problems established in terms of mathematical communication were evaluated, the abbreviations of mathematical concepts were made by considering the Turkish Language Association. Accordingly, the abbreviations in the measurement learning domain were based on m (meter), mm (millimeter), cm (centimeter), km (kilometer), ml (milliliter), l (liter), mg (milligram), g (gram), kg (kilogram), t (ton), min (minute), sec (second) and h (hour). Participant teachers were coded as "S1...S7" in the data analysis. The problems that were constructed according to the outcomes of the measurement learning domain of the primary school mathematics course in the PCT were expressed as V1 (calculating the perimeter lengths of the shapes), V2 (using time measurement units), V3 (using weighing units) and V4 (using liquid measurement units).

Reliability and Validity

In the process of reporting the research, the data were detailed clearly and consistently with each other to ensure credibility. In order to ensure transferability, the data were given in detail as they are. The consistency and confirmability of the research process plays an important role in the reliability of the research (Yıldırım & Şimşek, 2018). In order for the research results to be consistent and confirmable, the researchers tabulated the data according to the conceptual framework they created together. They confirmed each other throughout the analysis to ensure the consistency of their findings. Miles and Huberman (1994) formula (Reliability = agreement/agreement + disagreement) was used to calculate the coder reliability value. Coder consistency was calculated separately for each mathematical process skill. While the consistency in mathematical communication and association was one hundred percent, the consistency value in reasoning was found to be 0.92. Therefore, it can be stated that the study is reliable.

FINDINGS

Findings Related to the First Sub-Research Question

The problems posed by the teachers were analyzed according to their correct use of concepts, symbols and expressions and their use of different representations in terms of mathematical communication competencies and are shown in Table 1.

Table 1. Teachers' mathematical communication competence

Mathematical communication	Before Implementation Frequency	Frequency After Implementation

		V1	V2	V3	V4	V1	V2	V3	V4
According to the use of mathematical concepts, symbols and expressions	The right problem	5	4	4	4	7	6	5	6
	Incorrect/ Missing problem	2	3	3	3	-	1	2	1
Communication with different impressions	Verbal	7	7	7	7	5	7	7	7
	Graphic	-	-	-	-				
	Table	-	-	-	-				
	Image/Figure	-	-	-	-	2	-	-	-

According to Table 1, before the intervention, teachers used mathematical concepts, symbols and expressions correctly for 5 problems in V1, 4 in V2, 4 in V3 and 4 in V4, while after the intervention, this situation was 7 in V1, 6 in V2, 5 in V3 and 6 in V4. For V1 before the intervention, S2 did not specify the desired unit in the problem and S3 did not clearly express the shape whose perimeter was to be calculated, causing the problem to be incomplete in terms of mathematical communication. For V2 before the intervention, S2, S3 and S7 misrepresented the time. S2 also showed the abbreviation of minute incorrectly. For V3, S2, S3 and S6 made errors in the use of mathematical concepts, symbols and expressions before the application. S2, S6 used the concept of weight instead of mass. While S6 should have used the concept of kilogram, it was observed that she made a mistake by including the expression kilo, which is used colloquially, in the problem she set up. In S3's problem, it was not clear what and how many kg was asked due to the lack of mathematical expressions. Before the application for P4, S2 used the symbol for liter incorrectly as 'lt' and S5 used the symbol for milliliter incorrectly as 'mL'. In the problem posed by S6, there was a deficiency in terms of mathematical communication due to the inadequacy of the expression. The problem posed by S2 for V1 before the application is shown in Figure 2 and the problem posed by S6 for V3 is given as an example in Figure 3.

> A rectangular shaped astroturf field is surrounded by 3 rows of wire mesh. 180 m of wire mesh was used and when one of the sides of the field is 10 m, what is the length of the other side?

Figure 2. Example for V1 in terms of mathematical communication before implementation

> A viewer of the documentary Heavy Lives watched a patient who weighed half a ton and lost 300 kg in 2 years by undergoing a stomach reduction operation. Since the patient's goal is 80 kg, how many kilograms more should he lose?

Oriental Renaissance: Innovative, educational, natural and social sciences
Research BIB / Index Copernicus
(E)ISSN: 2181-1784
4(10), Nov., 2024
www.oriens.uz

Figure 3. Example for V3 in terms of mathematical communication before implementation

Before the application, teachers expressed the problems they constructed with verbal representation. After the intervention, S6 and S7 for V1 included visual/shape representation along with verbal representation. In Figure 4, the problem posed by S6 for V1 is shown as an example of communication with different representations.

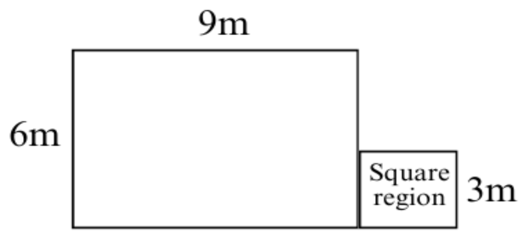	A part of the plot given in the figure is a quadrangle and the rest is a rectangle. If the perimeter of this plot is covered with 2 layers of wire, how many meters of wire is used?

Figure 4. Example for V1 in terms of communication with different representations after implementation

After the implementation, it was observed that teachers did not have any problems in terms of mathematical communication in V1. For V2, S3 did not specify the desired unit in the problem she constructed, which caused a deficiency in terms of mathematical communication. For V3, S1 and S6 used the concept of weight instead of mass in their problems. For V4, S2 used the milliliter symbol incorrectly. The problem that S1 constructed for V1 after the practice is given in Figure 5 as an example of mathematical communication.

The perimeter lengths of a square and a rectangle are equal to each other. The short side length of a rectangle is 6 cm and the long side length is 10 cm. According to these data, how many centimeters is the length of one side of the square?

Figure 5. Example for V1 in terms of mathematical communication after implementation

The problem that S4 set up for V2 after the application is shown in Figure 6 as an example of mathematical communication.

A train passed through the first tunnel in 3 minutes 45 seconds and the second tunnel in 4 minutes 38 seconds. How many seconds did the train cross the two tunnels in total?

Figure 6. Example for V2 in terms of mathematical communication after implementation

Findings Related to the Second Sub-Research Question

Teachers' mathematical association competencies were examined on the problems they constructed and shown in Table 2.

Table 2. Teachers' mathematical relatedness competence

Associating mathematics within itself	Before Implementation Frequency				Frequency After Implementation			
	V1	V2	V3	V4	V1	V2	V3	V4
There is an association between concepts.	-	6	5	6	5	5	3	3
No association between concepts.	7	1	2	1	2	2	4	4
There is an association between different representations.	-	-	-	-	2	-	-	-
No association between different representations.	7	7	7	7	5	7	7	7

According to Table 2, while classroom teachers made associations between concepts in 6 problems in V2, 5 problems in V3 and 6 problems in V4 before the implementation, they made associations in 5 problems in V1, 5 problems in V2, 3 problems in V3 and 3 problems in V4 after the implementation. In the problems for V2 before the application, S1, S2, S5 and S6 made associations between the concepts of minute-hour, while S4 and S7 made associations between the concepts of second-minute. In the problems for V3, S1, S2, S6 and S7 made associations between the concepts of kilogram-ton, S5 made associations between the concepts of gram-kilogram. In the problems for V4, S1, S2, S5, S6 and S7 made associations between the concepts of milliliter-liter and S4 made associations between the concepts of liter-pound. The problem that S2 constructed for V4 before the application is shown in Figure 7 as an example of the association between concepts.

Figure 7. Example for V4 in terms of associations between concepts before the

Zeynep bought 5 cases of soda for her birthday celebration. Each case contains 20 bottles and each bottle is 300 ml. How many liters of drinks were bought for the celebration?

implementation

S7's problem for V2 is shown in Figure 8 as an example of the association between concepts before the application.

In a soccer match that starts at 20:00, there is a 15-minute half-time break and 240 seconds of extra time is added to the first half and 300 seconds to the second half. Since both halves of the match lasted 45 minutes each, at what time did the match end?

Figure 8. Example for V2 in terms of associations between concepts before the implementation

In the problems for V1, S1, S4, S6 made associations between the concepts of square-rectangle, S7 made associations between square-rectangle-triangle, S3 made associations between the concepts of side lengths and triangle. In the problems for V2, S2, S5 made associations between h.-min. concepts, S4 between sec.-min. concepts, S6 between cm-m concepts, S7 between month-year concepts. In the problems for V3, S3 made associations between the concepts of kg-ton, S4, S7 made associations between the concepts of g-kg. In the problems for V4, S2, S4, S7 made associations between the concepts ml-l.

In Figure 9, the problem that S7 constructed for V3 after the implementation is shown as an example of conceptual association.

Ms. Sevgi puts 3 kg of spices in 50 g bags to sell in the market. Selling each bag of spices for 3 TL, how many TL does Mrs. Sevgi earn from these sales?

Figure 9. Example for V3 in terms of associations between concepts after the implementation

The problem posed by S2 for V4 after the application is shown in Figure 10 as an example of conceptual association.

A café buys 5 cases of milk daily. There are 20 bottles in each case and the volume of each bottle is 500 ml. According to this, how many liters of milk does this business buy daily?

Figure 10. Example for V4 in terms of associations between concepts after the implementation

While associations between different representations of concepts were not observed in the problems constructed before the intervention, they were observed only in the problems constructed by S6 and S7 for V1 after the intervention. The problem constructed by S7 is shown in Figure 11 as an example.

Ms. Senem ran 4 laps around a track consisting of square, rectangular and triangular areas with given dimensions. How many meters did Ms. Senem run?

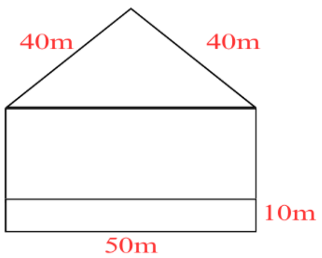

Figure 11. Example in terms of associations between different representations of concepts

Table 3. Teachers' competence in relating mathematics to different fields

Associating mathematics with different fields	Before Implementation Frequency				Frequency After Implementation			
	V1	V2	V3	V4	V1	V2	V3	V4
There is a connection to real life.	6	7	7	7	6	6	5	7
No real-life relevance.	1	-	-	-	1	1	2	-
There are associations with different disciplines.	-	-	-	-	-	1	2	-

According to Table 3, when the problems posed by the classroom teachers before the application were examined, it was seen that mathematics was associated

> The short side of a rectangle is 6 cm. Its long side is 4 less than 3 times the short side. What is the perimeter of this rectangle?

with real life in all problems except one problem for V1. While the classroom teachers did not associate mathematics with different disciplines before the application, after the application, S6 for V2 and S2 and S6 for V3 included associations with different disciplines in the problem. In the problem posed by S1 for V1 before the application, it was observed that mathematics was not associated with real life. This problem is given as an example in Figure 12.

Figure 12. Example of not associating mathematics with real life

The problem posed by S1 for V2 after the application is shown in Figure 13 as an example of associating mathematics with real life.

> Kaan starts class at 8:20 am and leaves the school after 4 hours and 20 minutes. At what time did Kaan leave the school?

Figure 13. Example of associating mathematics with real life

In Figure 14, the problem posed by S6 for P3 after the application is shown as an example of associating mathematics with different disciplines. Here, it can be stated that it is associated with the theme of 'Health and Sports' in the Turkish lesson.

Figure 14. Example of associating mathematics with different disciplines

> Naim Süleymanoğlu, who broke the world record by lifting 10 kg more than 3 times his own weight at the 1988 Summer Olympics, is 60 kg, how many kilograms did he lift?

Findings Related to the Third Sub-Research Question

Teachers' mathematical reasoning competencies were examined on the problems they constructed and shown in Table 4.

Table 4. Teachers' mathematical reasoning competence

Mathematical reasoning	Before Implementation Frequency				Frequency After Implementation			
	V1	V2	V3	V4	V1	V2	V3	V4
Mathematical reasoning based on creativity	-	-	-	-	1	-	1	1
Mathematical reasoning by analogy	7	7	7	7	6	7	6	6

According to Table 4, while the problem requiring creativity-based reasoning was not constructed before the intervention, it was constructed by S3 for V1 and V3 and S5 for V4 after the intervention.

The problem requiring reasoning based on analogy set by S5 for V4 before the implementation is shown in Figure 15.

How many liters of milk does a child who drinks 250 ml of milk every day drink in 20 days, and how many TL should he/she pay since milk costs 7 TL per liter?

Figure 15. Example of a problem requiring reasoning by analogy

In Figure 16, the problem posed by S3 for V3 after the implementation is given as an example of creativity-based reasoning.

A truck will be loaded with crates not exceeding 5 tons. 40 crates of 150 kg are loaded. However, the crates that are too much on the scale will be removed. At least how many crates should be removed?

Figure 16. Example of a problem requiring creativity-based reasoning

DISCUSSION AND CONCLUSION

Classroom teachers used mathematical concepts, symbols and expressions incorrectly/incompletely in the problems they constructed in the fourth grade measurement learning domain. There are some studies showing this situation in the literature (Hošpesová & Tichá, 2015; Kılıç, 2013; Serin, 2019; Tekin-Sitrava & Işık, 2018). Kılıç (2013) stated that pre-service primary school teachers had problems in expressing concepts related to fractions while constructing free problems. He associated this problem with the insufficiency of pre-service teachers' content knowledge. Hošpesová and Tichá (2015) stated that teachers and pre-service teachers had misconceptions about the concept of fraction in their problem posing research. Serin (2019) examined the problems constructed by prospective primary school teachers and stated that they were not sufficient in terms of mathematical language and expression. In addition, it was also observed that pre-service teachers constructed problems that were not suitable for the grade level. Sayın and Orbay (2024b) stated in their study that pre-service primary school teachers made mistakes in using mathematical concepts and symbols in the problems they constructed. After the problem-posing instruction, the pre-service teachers in the experimental group increased their level of correct use of mathematical expressions and stated that they constructed problems appropriate for the students' grade level. Sitrava and Işık (2018) stated that the insufficiency of pre-service primary school teachers' field knowledge and curriculum knowledge prevented them from constructing problems appropriate to the outcome. The problems seen in mathematical communication situations may stem from teachers' inadequate mathematics content knowledge. Problem posing can be used as a tool to evaluate teachers' mathematical concept knowledge. In addition, teachers' misuse of symbols may be due to inconsistencies in textbooks. For example, in the 4th grade science textbook (MoNE, 2021), the symbols of liquid measurement units are shown correctly, while in the mathematics textbook (MoNE, 2020) they are shown incorrectly.

After the problem posing instruction, the classroom teachers' levels of incorrect/incomplete use decreased and their mathematical communication competencies improved. In addition to verbal representation, they also used visual/shape representation in problems. However, teachers generally did not use different types of representations in problem posing. It can be thought that classroom teachers do not have sufficient knowledge about different types of representations because they do not use graphical and tabular representations. Similarly, pre-service primary school teachers generally used verbal representation (Ellerton, 2013; Kılıç, 2013) and did not use visual representation (Tekin-Sitrava & Işık, 2018). Işık, Işık, and Kar (2011) stated that when the problems constructed by pre-service teachers according to verbal representation and visual representation were compared, they showed lower success in constructing problems according to visual representation. The researchers explained this situation as interpreting the information in the visual representation and constructing problems requires advanced cognitive ability.

Before the problem posing instruction, classroom teachers were successful in establishing interconceptual associations in other problems except V1. After the intervention, there was an increase in interconceptual association for V1, while V2, V3 and V4 showed close results in interconceptual association. In the measurement learning domain, it was observed that teachers generally showed an average success in making associations between concepts and made associations between sub-concepts in the problems they constructed. No associations were made between different representations of the concept. Coşkun (2013) stated that while classroom teachers and mathematics teachers mostly made associations between concepts in their mathematics lessons, they did not make enough associations between different representations of the concept. After the implementation, associations between different concepts were made between figure and verbal representations.

Classroom teachers were successful in associating the problems they constructed with real life. Similarly, there are studies in which mathematics is associated with real life (Coşkun, 2013; Özgeldi & Osmanoğlu, 2017). In a study conducted by Özgeldi and Osmanoğlu (2017) on associating mathematics with real life, it was observed that pre-service teachers did not have any problems in associating mathematics with real life and stated that they were aware of the positive contribution of associating mathematics with real life on students. Coşkun (2013) stated that in mathematics lessons, teachers give a lot of space to real-life associations. Before the implementation, associations with different disciplines were not made. Aladağ and Şahinkaya (2013) examined the views of social studies and classroom teachers on making associations with different disciplines and found that although teachers know how to make associations with different disciplines, they do not know how to do it. Özdiner (2021) examined mathematics textbooks in basic education and found that associating with different disciplines was inadequate. Özgen (2019) stated that prospective physics, chemistry and mathematics teachers remained superficial in associating with different disciplines while preparing activities and stated that they were not at a sufficient level. After the implementation, classroom teachers made associations with different disciplines, albeit in small numbers. It was observed that the fact that the classroom teachers did not sufficiently associate different disciplines in the problems they constructed was similar to the related literature. This situation can be explained by the fact that the textbooks used by the teachers as a source are inadequate in terms of association skills, and that future teachers have theoretical knowledge in terms of association skills, but they have problems in practice.

In terms of mathematical reasoning, it was observed that the problems were based on analogy- based reasoning. After the application, the classroom teachers constructed problems requiring creativity- based reasoning, albeit in small numbers. When the literature is examined, it is seen that in problem posing studies, pre-service teachers have difficulty in posing non-routine problems that require creativity-based reasoning In general, pre-service teachers pose routine problems that are exercises requiring simple four operations (Crespo, 2003; Crespo & Sinclair, 2008; Işık & Kar, 2012; Korkmaz & Gür, 2006; Serin, 2019). Işık et al. (2011) stated that pre-service teachers construct problems that require simple solutions, can be solved with conventional solutions and do not require higher-order thinking. Xie and Masingila (2017) stated that pre-service teachers do not construct problems that require sufficient creativity. Çomarlı and Özdemir (2019) stated that far from constructing original problems, teachers construct free problems similar to those in the textbook. This situation can be explained by the fact that teachers' non-routine problem posing competencies that provide original creative thinking are not sufficient and they use similar problems by limiting themselves to the problems in the textbooks.

As a result, it was observed that the classroom teachers had problems in terms of mathematical communication in the problems they constructed, mostly did not use different forms of representation other than verbal representation, generally did not use different representations of the concept and made associations with different disciplines. In addition, it was determined that the majority of the problems they constructed required reasoning based on analogy and were simple and routine problems.

The problem posing test provides an opportunity to assess teachers' mathematical content knowledge in terms of using concepts. The conceptual framework created in the study to reveal mathematical process skills can be used as a tool to evaluate the quality of problems used in classrooms.

RECOMMENDATIONS

In mathematics teaching, problem posing activities can be used as a tool to reveal the participants' conceptual knowledge.

The conceptual framework used in the research can be used to reveal the quality of the problems used in mathematics teaching in terms of mathematical process skills.

Problem posing studies with classroom teachers or mathematics teachers can be conducted in different learning areas and at different grade levels in terms of mathematical process skills.

In textbooks, mathematical symbols can be used as specified by the Turkish Language Association to ensure accurate mathematical communication.

In-service trainings and projects can be carried out to improve classroom teachers' competencies in non-routine problem posing that requires reasoning based on creativity.

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