# INTEGRAL FORMULA FOR SPECIAL DERIVATIVES OF COMPLEX VARIABLE FUNCTIONS

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## ABSTRACT

*We know that integral formulas are used to solve a number of problems in mathematics and in other areas of our lives as well. In this paper, we have derived the integral formula for the special derivatives of the function when the complex variable satisfies the holomorphic condition on the n-1 variable and the Lipshits condition with the single variable.*

**Keywords:** *Holomorphic function, density point, asymptotic aspiration, Lipshits condition, differentiability.*

## АННОТАЦИЯ

*Мы знаем, что интегральные формулы используются для решения ряда задач в математике, а также в других областях нашей жизни. В этой статье мы вывели интегральную формулу для специальных производных функции, когда комплексная переменная удовлетворяет условию голоморфности для переменной n-1 и условию Липшица для единственной переменной.*

***Ключевые слова:*** *голоморфная функция, точка плотности, асимптотическое стремление, условие Липшица, дифференцируемость.*

## INTRODUCTION

Suppose we have a function f(z) in the field of $D$ complex field.

**Definition 1**. F function is believed to satisfy Lipshits contion in the field of D, If we have constant number L, for any of these points $z_1, z_2 \in D$ this inequality

$$\left|f(z_1) - f(z_2)\right| \le L\left|z_1 - z_2\right|$$

is reasonable.

In any $D \subset \mathbb{C}$ field we define set of functions that satisfy Lipshits condition as $Lip(D)$.

**Definition 2.** In $z \to z_0$, $f_{\bar{z}} \to 0$ is said to be asymptotic, if we have a set $Q(z_0)$ with $\lim\limits_{\substack{z \to z_0 \\ z \in Q(z_0)}} f_{\bar{z}} = 0$, in other words z point of $Q(z_0)$ set is aspired to the point $z_0$, ( normal) limit of $f_{\bar{z}}(z_0)$ is equal to zero.

Note that if there is a normal limit, there is an asymptotic limit, but vice versa is not always appropriate.

Any $B\{z \in \mathbb{C} : |z - z_0| < \varepsilon\}$ circle with the central point $z_0$ and the radius $r$, we denote these following:

$$\frac{\mu(B \cap Q)}{\mu(B)} = 1 - \delta(r), \qquad \frac{\mu(B \setminus Q)}{\mu(B)} = \delta(r),$$

here $\mu -$ is Lebeg measurement.

**Literature review and method**

Up to this day, we know integral formulas by Caushy, Martinelli-Boxner, Bergman-Weil and Lere. These formulas were included in litetatures [1,7]. Using these literatures, we have derived integral formula for special derivatives of complex variable functions. We used Theorem 1 and Theorem 2 to derive this formula. Caushy formula was used to prove it.

**Theorem 1.** If $f \in Lip(d)$ and $z_0 \in D$. If in $z \to z_0$, $f_{\bar{z}} \to 0$ is asmptotic, then the inequality

$$\left| \frac{f(z_0 + h) - f(z_0)}{h} - \frac{1}{2\pi i} \int_{\partial B} \frac{f(\zeta)d\zeta}{(\zeta - z_0)^2} \right| \le$$
$$\le \varepsilon(r) M_1(r, \zeta) + 2LM_2(r, \zeta)$$

is reasonable.

Here when $\varepsilon(r) = \sup p|f_{\bar{z}}|$ is true, $M_1$ and $M_2$ are constants related to $r$ and $h$.

**Theorem 2.** If $f \in Lip(D)$, $z_0$ is any point of field $D$. If $z \to z_0$, $f_{\bar{z}} \to 0$ is asymptotic, then funtion f in $z_0$ point, $\mathbb{C}$ is differentiation.

**RESULTS**

From the conditions of Theorem 2, the formula $f'(z_0) = \lim\limits_{r \to 0} \frac{1}{2\pi i} \int_{\partial B} \frac{f(\zeta)d\zeta}{(\zeta - z_0)^2} = \frac{\partial f(z_0)}{\partial \bar{z}}$ is reasonable.

**Theorem( main).** Suppose that a function $f(z)=f(z_1,...,z_n)$ in the field of $D\subset\mathbb{C}^n$ with variables $z_1,z_2,...z_{k-1},z_{k+1},...,z_n$ is holomorphic and satisfies the Lipshits condition on the variable $z_k$, $a\,(a\in D\;\; a=(a_1,...,a_n))$ is any point in the field.

If $z_k\to a_k$, $\frac{\partial f}{\partial \bar{z}_k}\to 0$ is asypmtotic, then the formula

$$\frac{\partial f(a)}{\partial z_k}=\lim_{r_k\to 0}\frac{1}{(2\pi i)^n}\int_{\Gamma}\frac{f(\zeta_1,...,\zeta_n)d\zeta_1,...,d\zeta_n}{(\zeta_1-a_1)(\zeta_2-a_2)...(\zeta_k-a_k)^2...(\zeta_n-a_n)}$$

is appropriate.

Here $\Gamma-H(a,r)\subset D$ is a semicircle .

**DISCUSSION**

**Proof**. Suppose that the semicircle $U=\{z\in C^n:|z_v-a_v|<r_v,\, v=1,2,...,n\}$ with the centre a and the radius $r=(r_1,...,r_k)$ is on $D$.

According to the Theorem 2 the equality

$$\frac{\partial f(a)}{\partial z_n}=\lim_{r_n\to 0}\frac{1}{2\pi i}\int_{\Gamma_n}\frac{f(a',\zeta_n)d\zeta_n}{(\zeta_n-a_n)^2}$$

is appropriate. Since

$z_1,z_2,...,z_{n-1}$ is a holomorphic function with complex variables, the formula $f(''\zeta,\zeta_{n-1},\zeta_n)=\frac{1}{2\pi i}\int_{\Gamma_{n-1}}\frac{f(''\zeta,\zeta_{n-1},\zeta_n)d\zeta_{n-1}}{(\zeta_{n-1}-a_{n-1})}$

is appropriate, if we use Caushy formula

$n-1$ times, the following integral is derived:

$$\frac{\partial f(a)}{\partial z_k} = \lim_{r_n \to 0} \frac{1}{(2\pi i)^n} \int_{\Gamma_n} \int_{\Gamma_{n-1}} \dots \int_{\Gamma_1} \frac{f(\zeta_1, \zeta_2, \dots, \zeta_n) d\zeta_n d\zeta_{n-1} \dots d\zeta_1}{(\zeta_n - a_n)^2 (\zeta_{n-1} - a_{n-1}) \dots (\zeta_1 - a_1)} =$$

$$= \lim_{r_n \to 0} \frac{1}{(2\pi i)^n} \int_{\Gamma} \frac{f(\zeta_1, \dots, \zeta_n) d\zeta_1 \dots d\zeta_n}{(\zeta_1 - a_1) \dots (\zeta_{n-1} - a_{n-1})(\zeta_n - a_n)^2}$$

According to this, the theorem is proven.

## CONCLUSION

We may conclude by saying that up to this day many scientists have derived integral formulas for complex variable functions. The most generalized of all is the Caushy formula. In this paper, we derived integral formula( on the basis of theorem) not for the function itself but for it's special derivatives:

$$\frac{\partial f(a)}{\partial z_k} = \lim_{r_k \to 0} \frac{1}{(2\pi i)^n} \int_{\Gamma} \frac{f(\zeta_1, \dots, \zeta_n) d\zeta_1, \dots, d\zeta_n}{(\zeta_1 - a_1)(\zeta_2 - a_2) \dots (\zeta_k - a_k)^2 \dots (\zeta_n - a_n)}.$$

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