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EULER'S METHOD

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ABSTRACT

Up to this point practically every differential equation that we've been presented with could be solved. The problem with this is that these are the exceptions rather than the rule. The vast majority of first order differential equations can't be solved. In order to teach you something about solving first order differential equations we've had to restrict ourselves down to the fairly restrictive cases of linear, separable, or exact differential equations or differential equations that could be solved with a set of very specific substitutions.

Keywords: *differential equation, specific substitutions, explicit solutions, differential equation, Euler's Method. , IVP, Intervals of Validity.*

АННОТАЦИЯ

До сих пор можно было решить практически каждое дифференциальное уравнение, с которым нам приходилось сталкиваться. Проблема в том, что это скорее исключения, чем правило. Подавляющее большинство дифференциальных уравнений первого порядка решить невозможно. Чтобы научить вас решению дифференциальных уравнений первого порядка, нам пришлось ограничиться довольно ограниченными случаями линейных, сепарабельных или точных дифференциальных уравнений или дифференциальных уравнений, которые можно было решить с помощью набора очень специфических подстановок.

Ключевые слова: *дифференциальное уравнение, частные замены, явные решения, дифференциальное уравнение, метод Эйлера. , ИВП, Интервалы действия.*

INTRODUCTION

Most first order differential equations however fall into none of these categories. In fact, even those that are separable or exact cannot always be solved for an explicit solution. Without explicit solutions to these it would be hard to get any information about the solution.

So, what do we do when faced with a differential equation that we can't solve? The answer depends on what you are looking for. If you are only looking for long term behavior of a solution you can always sketch a direction field. This can be done

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without too much difficulty for some fairly complex differential equations that we can't solve to get exact solutions.

DISCUSSION AND RESULTS

The problem with this approach is that it's only really good for getting general trends in solutions and for long term behavior of solutions. There are times when we will need something more. For instance, maybe we need to determine how a specific solution behaves, including some values that the solution will take. There are also a fairly large set of differential equations that are not easy to sketch good direction fields for.

In these cases, we resort to numerical methods that will allow us to approximate solutions to differential equations. There are many different methods that can be used to approximate solutions to a differential equation and in fact whole classes can be taught just dealing with the various methods. We are going to look at one of the oldest and easiest to use here. This method was originally devised by Euler and is called, oddly enough, Euler's Method.

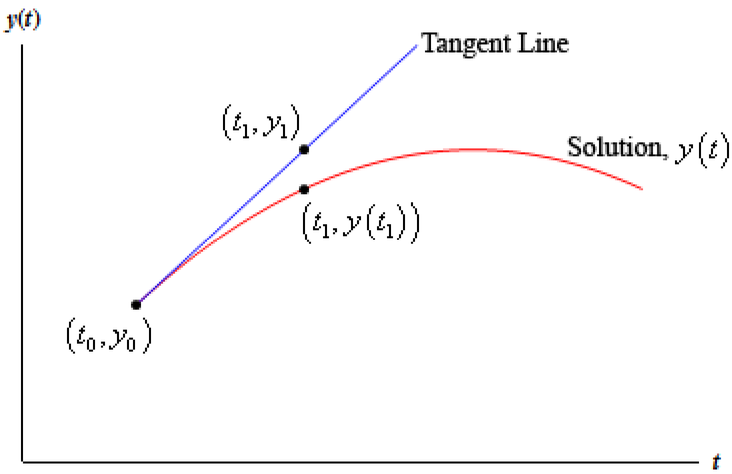
Let's start with a general first order IVP

Dy/dt=f(t,y)   y(t0)=y0

where f(t,y)f(t,y) is a known function and the values in the initial condition are also known numbers. From the second theorem in the Intervals of Validity section we know that if ff and fyfy are continuous functions then there is a unique solution to the IVP in some interval surrounding t=t0t=t0. So, let's assume that everything is nice and continuous so that we know that a solution will in fact exist.

dydt|||t=t0=f(t0,y0)dydt|t=t0=f(t0,y0) Now, recall from your Calculus I class that these two pieces of information are enough for us to write down the equation of the tangent line to the solution

y=y0+f(t0,y0)(t−t0)y=y0+f(t0,y0)(t−t0) Take a look at the figure below



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If t1t1 is close enough to t0t0 then the point y1y1 on the tangent line should be fair easy enough. All we need to do is plug t1t1 in the equation for the tangent line.

$y1=y0+f(t0,y0)(t1-t0)y1=y0+f(t0,y0)(t1-t0)$

Now, we would like to proceed in a similar manner, but we don't have the value of the solution at t1t1 and so we won't know the slope of the tangent line to the solution at this point. This is a problem. the solution and construct a line through the point (t1,y1t1,y1) that has slope f(t1,y1f(t1,y1). This gives

$y=y1+f(t1,y1)(t-t1)y=y1+f(t1,y1)(t-t1)$

Now, to get an approximation to the solution at t=t2t=t2 we will hope that this new

computed approximation to get the next approximation. So,

$y3=y2+f(t2,y2)(t3-t2)y4=y3+f(t3,y3)(t4-t3)etc.y3=y2+f(t2,y2)(t3-t2)y4=y3+f(t3,$

$y3)(t4-t3)etc.$

If we define fn=f(tn,yn)fn=f(tn,yn) we can simplify the formula to

$yn+1=yn+fn\cdot(tn+1-tn)(2)(2)yn+1=yn+fn\cdot(tn+1-tn)$

This doesn't have to be done and there are times when it's best that we not do this. However, if we do the formula for the next approximation becomes.

$yn+1=yn+hfn(3)(3)yn+1=yn+hfn$

So, how do we use Euler's Method? It's fairly simple. We start with (1)(1) and decide if we want to use a uniform step size or not. Then starting with (t0,y0)(t0,y0) we repeatedly evaluate (2)(2) or (3)(3) depending on whether we chose to use a uniform step size or not. We continue until we've gone the desired number of steps or reached the desired time. What do we do if we want a value of the solution at some other point than those used here? One possibility is to go back and redefine our set of points to a new set that will include the points we are after and redo Euler's Method using this new set of points. However, this is cumbersome and could take a lot of time especially if we had to make changes to the set of points more than once.

Another possibility is to remember how we arrived at the approximations in the first place. Recall that we used the tangent line

$y=y0+f(t0,y0)(t-t0)y=y0+f(t0,y0)(t-t0)$

to get the value of y1y1. We could use this tangent line as an approximation for the

solution on the interval [t0,t1][t0,t1]. Likewise, we used the tangent line

$y=y1+f(t1,y1)(t-t1)y=y1+f(t1,y1)(t-t1)$

In practice you would need to write a computer program to do these compu tations for you. In most cases the function f(t,y)f(t,y) would be too large and/or

complicated to use by hand and in most serious uses of Euler's Method you would want to use hundreds of steps which would make doing this by hand prohibitive.

CONCLUSION

So, Euler's method is a nice method for approximating fairly nice solutions that don't change rapidly. However, not all solutions will be this nicely behaved. There are other approximation methods that do a much better job of approximating solutions. These are not the focus of this course however, so I'll leave it to you to look further into this field if you are interested. Also notice that we don't generally have the actual solution around to check the accuracy of the approximation. We generally try to find bounds on the error for each method that will tell us how well an approximation should do. These error bounds are again not really the focus of this course, so I'll leave these to you as well if you're interested in looking into them.

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