

## **VISCOELASTIC PLASTIC MECHANICAL SYSTEMS WITH SPOT LINKS AND THEIR OWN VIBRATIONS**

**Saliyeva Olima Kamalovna**

Candidate of technical Sciences, associate Professor, department of “Information-communication systems of controlling technological processes” Bukhara engineering-technological institute, Bukhara city, Uzbekistan, [saliyevaok@mail.ru](mailto:saliyevaok@mail.ru)

**Sharipova Nazira Rakhmatilloevna**

Assistant, department of “Information-communication systems of controlling technological processes” Bukhara engineering-technological institute, Bukhara city, Uzbekistan

### **ABSTRACT**

*The article presents the natural oscillations of viscoelastic lamellar mechanical systems with point connections are considered. Frequency equations are obtained and solved numerically by the Muller method. A parametric analysis of complex eigenfrequencies depending on the geometric parameters is given.*

**Keywords:** *Free oscillations, dissipative system, vibrations, viscoelastic system.*

### **АННОТАЦИЯ**

*В статье рассматриваются собственные колебания вязкоупругих пластинчатых механических систем с точечными соединениями. Получены частотные уравнения и численно решены методом Мюллера. Дан параметрический анализ комплексных собственных частот в зависимости от геометрических параметров.*

**Ключевые слова:** *Свободные колебания, диссипативная система, колебания, вязкоупругая система.*

### **INTRODUCTION**

The structural inhomogeneity of the system is determined by the presence in it of viscoelastic elements with different dissipative properties (otherwise, it is a structurally homogeneous viscoelastic system). A mechanical system here means a rectangular plate, a package of rectangular plates, a shell of revolution, a system of shells of revolution having point connections. Free oscillations of a dissipative system are damped. The amplitudes of the oscillation modes decrease with time, so such a process, strictly speaking, is not periodic. But the frequencies of the corresponding forms remain constant [1, 2] and, in this sense, a dissipative system can be studied as a system with natural oscillations.

**Formulation of the problem.** Consider a mechanical system consisting of  $N$  isotropic viscoelastic bodies occupying a volume  $V_n$  or limited surfaces  $\Omega_n$  ( $n = 1, \dots, N$ ). It is assumed that one linear dimension of each body is much smaller than the other two. For each  $n$ , on the part of the surface of the  $n$ th body,  $\Omega_n^{sb}$  homogeneous boundary conditions, on the rest of the free surface  $\Omega_n^{bs} = \Omega_n / \Omega_n^{sb}$  kinematic and dynamic bonds are imposed at a finite number of points: point rigid, elastic and (or) viscoelastic hinge-type supports (rigid supports can be pinched), rigid elastic and (or) viscoelastic shock absorbers connecting the bodies (when  $N > 1$ ), concentrated masses  $M_{qn}$  ( $q = 1, \dots, Q$ ). Расположение связей и масс на поверхностях  $\Omega_n^{bs}$  произвольно.

In the general case, the dissipative properties of the system elements are different. A special case of such a structurally inhomogeneous viscoelastic system is a system with elastic and viscoelastic elements. For the last case  $N = N_y + N_n$ , where  $N_y$  – number of elastic elements of the system,  $N_n$  – the number of viscoelastic elements. Under  $N > 2$  bodies are parallel to each other with free surfaces  $\Omega_n^{bs}$  (shell plate packs). When  $N=1$ , there are no racks. It is required to determine the natural frequencies of the viscoelastic system, as well as to evaluate its damping capacity. In the mathematical formulation, viscoelasticity looks like this. Let all points of the  $n$ th body obey the harmonic law of oscillations.

$$U_{nj}(\bar{x}^n, t) = U_{nj}^0(\bar{x}^n) e^{-i\omega t}, \quad n = 1, \dots, N, j = 1, \dots, J, \quad (1)$$

where  $U_{nj}^0(\bar{x}^n)$  –  $j$ - component of displacement vector  $n$ -body,  $J$ - number of displacement vector components,  $\bar{x}^n = (x_1^n, x_2^n, x_3^n)$  – radius-vector of the point of the  $n$ -th body,  $\omega = \omega_R + i\omega_I$  – desired complex frequency of the system, and  $\omega_R$  – natural frequency, a  $\omega_I$  – damping factor ( $\omega_I < 0$ ). Since each component of the displacement vector already has an index  $n$ , the latter is not used to designate the components of the radius vector in what follows.

For rectangular inserts  $J = 1$  and

$$U_{n1}^0(x_1, x_2) = W_n^0(x, y),$$

for shells of revolution  $J = 3$  or

$$U_{n1}^0(x_1, x_2) = U_n^0(x, y), U_{n2}^0(x, y) = V_n^0(x, y), U_{n3}^0(x, y) = W_n^0(x, y),$$

where  $x, y$  – coordinates. Based on the principle of possible displacements, we equate to zero the sum of the work of all active forces, including the forces of inertia on possible displacements  $\delta U_{nj}(\bar{x}, t)$ :

$$\delta A_\sigma + \delta A_a + \delta A_m = 0, \quad (2)$$

where  $\delta A_\sigma, \delta A_a, \delta A_m$  – virtual work of the internal forces of the bodies of springs, as well as the forces of inertia, taking into account concentrated masses. These works can be represented by the following relations:

$$\begin{aligned} \delta A_\sigma &= - \sum_{n=1}^N \sigma_{mk}^n \delta \varepsilon_{mk}^n dV, \\ \delta A_a &= - \sum_{n=1}^N \sum_{l=1}^{L_n} \sigma_l^n \delta \varepsilon_l^n - \sum_{n=1}^N \sum_{l'=1}^{L'_n} \sigma_{l'}^n \delta \varepsilon_{l'}^n, \\ \delta A_m &= \\ &- \sum_{n=1}^N \rho_n \int_{V_n} \left( \sum_{j=1}^J \ddot{U}_{nj}(\bar{x}, t) \delta U_{nj} \right) dV - \sum_{n=1}^N \sum_{q=1}^{Q_n} M_{qn} \sum_{j=1}^J \ddot{U}_{nj}(\bar{x}_n^q, t) \delta U_{nj}, \end{aligned} \quad (3)$$

where  $\rho_n, V_n$  – density and volume of the  $n$ th body,  $M_{qn}$  –  $q$ -th attachment mass of the  $n$ -th body with coordinates

$$\bar{x}_n^q = (x_{n1}^q, x_{n2}^q, 3), L_n$$

- the number of springs (shock absorbers) between  $n$ - $m$  и  $(n+1)$ -  $m$  bodies,  $Q_n$  – number of concentrated masses on the  $n$ th body,  $L'_n$  – number of elastic (viscoelastic) supports on the  $n$ th body,  $\sigma_{mk}^n, \varepsilon_{mk}^n, \sigma_l^n, \varepsilon_l^n, \sigma_{l'}^n, \varepsilon_{l'}^n$  – components of stress and strain tensors, respectively, of the  $n$ th body, the  $l$ -th spring (shock absorber) and  $l'$ -th elastic (viscoelastic) support.

Physical and geometric relationships for an elastic element or an elastic connection of a system can be written using the generalized Hooke's law

$$\sigma_{mk}^n(t) = \tilde{\lambda}_n \Theta^n(t) \delta_{mk} + 2\tilde{\mu}_n \varepsilon_{mk}^n(t),$$

where  $\tilde{\lambda}_n, \tilde{\mu}_n$  – Volterra integral operators, which are replaced by a single operator below. Expressing  $\tilde{\lambda}_n, \tilde{\mu}_n$  according to known formulas  $\tilde{E}_n, \tilde{\nu}_n$  and given that  $\tilde{\nu}_n = \nu_n = const$ , where

$$(\tilde{E}_n \varphi)(t) = E_n \left[ \varphi(t) - \int_0^t R^n(t - \tau) \varphi(\tau) d\tau \right], \quad (4)$$

here  $E_n$  – instant modulus of elasticity, a  $R^n$  – relaxation core.

Considering (1), the function of time in equality (4) will be  $\varphi(t) = \exp(-i\omega t)$  with slowly changing amplitude. Assuming the smallness of the integral  $\int_0^\infty R(\tau) d\tau$ , using the freezing method, we replace relation (4) with an approximate one:

$$\tilde{E}_n \cong E_n [1 - \Gamma_c(\omega_R) - is(\omega_R)] \varphi, \text{ where}$$

$$\left\{ \begin{matrix} \Gamma_c \\ \Gamma_s \end{matrix} \right\} = \int_0^\infty R^n(\tau) \left\{ \begin{matrix} \cos \omega_R \tau \\ \sin \omega_R \tau \end{matrix} \right\} d\tau.$$

This makes it possible to exclude integral terms and, ultimately, time from the variational equation. Symbolically, it can be represented as

$$\delta G(U_{nj}^0(\bar{X}), \omega^2) = 0. \quad (5)$$

If the n-th plate, l-th springy and l'-th viscoelastic support, then  $\bar{D}_n, \bar{C}_{ln}, \bar{C}_{l'/n}$  are represented by the following formulas:

$$\bar{D}_n = D_n f_n(\omega_R), \quad \bar{C}_{ln} = C_{ln} f_{ln}(\omega_R), \quad \bar{C}_{l'/n} = C_{l'/n} f_{l'/n}(\omega_R),$$

where

$$f(\omega_R) = 1 - \Gamma_c(\omega_R) - i\Gamma_s(\omega_R)$$

- complex function whose numerical coefficients depend on the parameters of the relaxation kernel of the corresponding viscoelastic elements,

$$D_n = \frac{E v h_n^3}{12(1-v_n^2)}, C_{ln}, C_{l'/n} -$$

generalized instantaneous stiffnesses, respectively, of the n-th plate, l-th shock absorber, l'-th supports. In the elastic case  $\bar{D}_n = D_n, \bar{C}_{ln} = C_{ln}, \bar{C}_{l'/n} = C_{l'/n}$ , where  $D_n, C_{ln}, C_{l'/n}$  - generalized stiffness's of the n-th plate, l -th spring, l'-th support, respectively.

It is necessary to find the spectrum of complex natural frequencies

$$\omega^k = \omega_R^k + i\omega_I^k,$$

where  $\omega_R^k$  - frequencies, and  $\omega_I^k$  - damping coefficients of own oscillation damping.

**Numerical results.** Consider a structure that is a package of two parallel square elastic plates with a shock absorber and an attached mass. The relaxation kernel for the shock absorber has chosen in the form

$$R(t) = A \exp(-\beta t) t^{\alpha-1},$$

where  $A, \beta, \alpha$  - kernel parameters [2].

The viscosity of the shock absorber is taken such that its creep deformation during the quasi-static process is a small fraction of the total (~12%). For this case, the kernel parameters are as follows:  $A = 0.01, \alpha = 0.1, \beta = 0.05$  [2].

In contrast to the elastic problem, here we studied the dependence of two low frequencies and the corresponding damping coefficients on the value of the instantaneous stiffness of the shock absorber. The latter has changed from  $10^{-4}$  to  $10^{-1}$ . On the right, this range is limited by the value, since at  $C=C_2$  there is a change of the second form. On fig. 1 shows the dependence of the first two frequencies  $\omega_R^1, \omega_R^2$  and corresponding damping coefficients  $\omega_I^1, \omega_I^2$  on the value of the instantaneous stiffness of the shock absorber C. From the analysis of the graphs, it follows that the dissipative properties of this system as a whole are determined not only by the rheology of its elements, but significantly depend on the interaction of oscillations of natural forms.

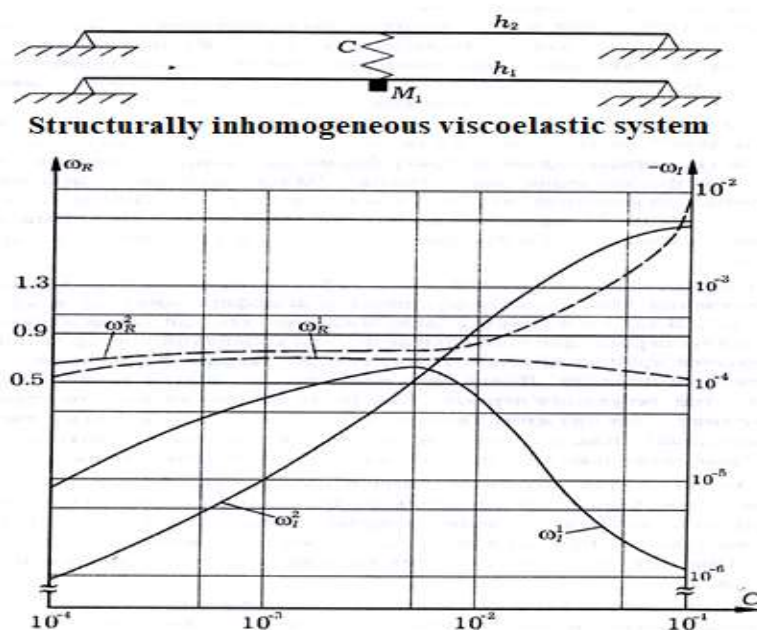


Figure 1. Dependence of frequencies and damping coefficients

This effect is expressed in the fact that under certain conditions (about them below) and up to a certain value of the shock absorber stiffness, the energetically more capacious (in this case, the second) form dissipates less energy than the less energy-intensive form. Then, starting from some value of the instantaneous stiffness of the shock absorber (in this case, from  $C^* = 5.4 \cdot 10^{-3}$ ), the process of dissipation of energy by its own forms is normalized and proceeds according to the energy hierarchy of forms.

### CONCLUSION

The practical conclusion is as follows: the damping capacity of the structure is mainly determined by the minimum absolute value of the damping coefficient (in this case, oscillations of this particular form are the last to damp); the global (defining) damping coefficient of the system is first  $\omega_I^1$  to the point of intersection, and then  $\omega_I^2$ . Optimal in terms of attenuation, the vibration mode of the structure will be at  $C=C^*$ , when this global damping factor is at its maximum.

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