CHIZIQLI TENGLAMALAR SISTEMASI VA UNI YECHISHNING TURLI USULLARI

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ANNOTATSIYA

Mazkur maqolada chiziqli tenglamalar sistemasi tushunchasi, ularning matematik model sifatidagi ahamiyati hamda yechish usullari yoritilgan. Xususan, sistemani matritsaviy ko'rinishda ifodalash, koeffitsiyentlar matritsasi, noma'lumlar ustuni va ozod hadlar ustuni tushunchalari izohlangan. Shuningdek, chiziqli tenglamalar sistemasini yechishda determinant, teskari matritsa va Gauss usuli asosida bajariladigan bosqichlar tahlil qilingan. Maqolada matritsaviy usulning nazariy asoslari bilan bir qatorda, uni amaliy masalalarni yechishda qo'llash imkoniyatlari ham ko'rsatib berilgan. Sistemalarning yagona yechimga ega bo'lishi, cheksiz ko'p yechimga ega bo'lishi yoki yechimga ega bo'lmaslik holatlari matritsa rangi va determinant qiymatiga bog'liq holda tushuntirilgan. Tadqiqot natijalari chiziqli tenglamalar sistemasini samarali va ixcham usulda yechishda matritsaviy yondashuv muhim ahamiyatga ega ekanligini ko'rsatadi.

Kalit so'zlar: *Gauss usuli, chiziqli tenglamalar sistemasi, Kramer usuli, matritsaviy usul, determinant.*

ABSTRACT

This article discusses the concept of a system of linear equations, their importance as a mathematical model, and methods for solving them. In particular, the concepts of expressing the system in matrix form, the matrix of coefficients, the column of unknowns, and the column of free terms are explained. The steps performed in solving a system of linear equations based on the determinant, inverse matrix, and Gaussian method are also analyzed. The article, along with the theoretical foundations of the matrix method, also shows the possibilities of its application in solving practical problems. The cases of systems having a unique solution, having infinitely many solutions, or not having a solution are explained depending on the color of the matrix and the value of the determinant. The results of *the study show that the matrix approach is important in solving a system of linear equations in an efficient and compact way.*

***Keywords**: Gaussian method, system of linear equations, Cramer's method, matrix method, determinant.*

KIRISH

Hozirgi kunda matematika fanining muhim yo‘nalishlaridan biri bo‘lgan chiziqli algebra nafaqat nazariy jihatdan, balki amaliy masalalarni yechishda ham katta ahamiyat kasb etmoqda. Ayniqsa, chiziqli tenglamalar sistemasi va ularni samarali yechish usullari ilm-fan, muhandislik, iqtisodiyot hamda axborot texnologiyalari sohalarida keng qo‘llanilmoqda. Chiziqli tenglamalar sistemalarini yechish muammosi qadimdan matematiklarning e’tibor markazida bo‘lib kelgan va bu borada turli usullar ishlab chiqilgan.

Zamonaviy yondashuvlar ichida matritsaviy usul alohida o‘rin egallaydi. Ushbu usul chiziqli tenglamalar sistemasini ixcham ko‘rinishda ifodalash, hisoblash jarayonlarini soddalashtirish va kompyuter yordamida tezkor yechim olish imkonini beradi. Matritsalar yordamida tenglamalar sistemasini ifodalash orqali muammoni umumlashtirish, algoritmlashtirish va dasturlash imkoniyatlari kengayadi. Bu esa ayniqsa raqamli texnologiyalar rivojlangan hozirgi davrda muhim ahamiyat kasb etadi.

Mazkur maqolada chiziqli tenglamalar sistemasining asosiy tushunchalari, ularni matritsalar yordamida ifodalash va matritsaviy usullar orqali yechish mexanizmlari tahlil qilinadi. Shuningdek, ushbu usulning afzalliklari, amaliy qo‘llanishi va ta’lim jarayonidagi o‘rni yoritiladi. Tadqiqot davomida nazariy bilimlar bilan bir qatorda amaliy misollar asosida masalaning mohiyati ochib beriladi.

1. Gauss usuli-noma‘lumlarni biri tanlanib uni yo‘qotib yuborish orqali ikkinchisini topilib, olingan natijani ixtiyoriy bir tenglamaga qo‘yish orqali natijaga erishiladi.

1-Ta‘rif: n noma‘lumli m ta ***chiziqli tenglamalar sistemasi*** deb quyidagi ko‘rinishdagi sistemaga aytiladi.

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \cdots + a_{2n}x_n = b_2 \\ \cdots \\ a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \cdots + a_{mn}x_n = b_m \end{cases}$$

bu yerda a_{11} – koeffitseyentlar b_i – ozod had deyiladi. Bizga 2 noma‘lumli chiziqli tenglama berilgan bo‘lsin:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 = b_1 \\ a_{21}x_1 + a_{22}x_2 = b_2 \end{cases}$$

$$\begin{cases} a_{11}a_{21}x_1 + a_{12}a_{21}x_2 = b_1 \\ a_{11}a_{21}x_1 + a_{11}a_{22}x_2 = b_2 \end{cases}$$

$$a_{12}a_{21}x_2 - a_{11}a_{22}x_2 = b_1a_{21} - b_1a_{21}$$

$$x_2 = \frac{b_1a_{21} - b_1a_{21}}{a_{12}a_{21} - a_{11}a_{22}}$$

shu tartibda bir qiymati topilib, ikkinchi qiymati birinchisini o‘rniga qo‘yish orqali topiladi.

1-misol: $\begin{cases} 2x - 3y = 7 \\ x + 4y = 9 \end{cases}$ ushbu tenglamalar sistemasini Causs usuli yordamida yeching.

$$-11y = -11$$

$$y = 1, \qquad x = 9 - 4 = 5$$

2-misol:

$$\begin{cases} x + 2y + 3z = 2 \\ 2x + 3y - 4z = -5 \\ 3x + y + z = 3 \end{cases}$$

$$\begin{cases} x + 2y + 3z = 2 \\ 2x + 3y - 4z = -5 \\ -9x - 3y - 3z = -9 \end{cases}$$

$$-8x - y = -7$$

$$y = -8x + 7$$

$$x - 16x + 14 + 3z = 2$$

$$-15x + 3z = -12$$

$$z = 5x - 4$$

$$2x - 24x + 21 - 20x + 16 = -5$$

$$-42x = -42$$

$$x = 1; \qquad y = 1; \; z = 1.$$

2. Kramer usuli: 1-tenglamalar sistemasining yechimi asosiy va yordamchi determinantlar orqali ifodalanuvchi tengliklar Kramer usuli deb aytiladi. Unda biz quyida keltiradigan asoslar bilan natijaga erishishni ko‘rib o‘tamiz. U uchun bizga 2 noma‘lumli chiziqli bog‘liq tenglamalar sistemasi berilgan bo‘lsin.

$$\begin{cases} a_{11}x_1 + a_{12}x_2 = b_1 \\ a_{21}x_1 + a_{22}x_2 = b_2 \end{cases}$$

Yuqorida berilganlardan quyidagi natijalarni olishimiz mumkin. Agar determinantning natijasi nolga teng bolsa, yechimga ega emas.

$$\Delta = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = a_{11} \cdot a_{22} - a_{12} \cdot a_{21}$$

$$\Delta x_1 = \begin{bmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{bmatrix} = b_1 \cdot a_{22} - a_{12} \cdot b_2$$

$$\Delta x_2 = \begin{bmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{bmatrix} = a_{11} \cdot b_2 - b_1 \cdot a_{21}$$

$$x_1 = \frac{\Delta x_1}{\Delta}; \qquad x_2 = \frac{\Delta x_2}{\Delta}$$

3-misol:

$\begin{cases} 2x - 3y = 7 \\ x + 4y = 9 \end{cases}$ ushbu tenglamalar sistemasini Kramer usulida yeching.

$$\Delta = \begin{bmatrix} 2 & -3 \\ 1 & 4 \end{bmatrix} = 2 \cdot 4 - (-3) \cdot 1 = 11$$

$$\Delta x_1 = \begin{bmatrix} 7 & -3 \\ 9 & 4 \end{bmatrix} = 7 \cdot 4 - (-3) \cdot 9 = 55$$

$$\Delta x_2 = \begin{bmatrix} 2 & 7 \\ 1 & 9 \end{bmatrix} = 2 \cdot 9 - 1 \cdot 7 = 11$$

$$x_1 = \frac{\Delta x_1}{\Delta} = \frac{55}{11} = 5; \qquad x_2 = \frac{\Delta x_2}{\Delta} = \frac{11}{11} = 5$$

Endi bizga 3 noma‘lumli chiziqli bog‘liq tenglamalar sistemasi berilgan bo‘lsin.

$$\Delta = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \neq 0$$

$$\Delta x_1 = \begin{bmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{bmatrix}$$

$$\Delta x_2 = \begin{bmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{bmatrix}$$

$$\Delta x_3 = \begin{bmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{bmatrix}$$

$$x_1 = \frac{\Delta x_1}{\Delta}; \qquad x_2 = \frac{\Delta x_2}{\Delta}; \qquad x_3 = \frac{\Delta x_3}{\Delta}$$

Mana shu tartibda noma‘lumlarni topib olamiz.

4-misol:

$$\begin{cases} x + 2y + 3z = 2 \\ 2x + 3y - 4z = -5 \\ 3x + y + z = 3 \end{cases}$$

$$\Delta = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & -4 \\ 3 & 1 & 1 \end{bmatrix} = 3 + 6 - 24 - 27 - 4 + 4 = -42;$$

$$\Delta x_1 = \begin{bmatrix} 2 & 2 & 3 \\ -5 & 3 & -4 \\ 3 & 1 & 1 \end{bmatrix} = 6 - 15 - 24 - 27 + 10 + 8 = -42;$$

$$\Delta x_2 = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -5 & -4 \\ 3 & 3 & 1 \end{bmatrix} = -5 + 18 - 24 + 45 - 4 + 12 = 42;$$

$$\Delta x_3 = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 3 & -5 \\ 3 & 1 & 3 \end{bmatrix} = 9 - 4 - 30 - 18 - 12 + 5 = -42;$$

$$x_1 = \frac{\Delta x_1}{\Delta} = \frac{-42}{-42} = 1;$$

$$x_2 = \frac{\Delta x_2}{\Delta} = \frac{42}{-42} = -1;$$

$$x_3 = \frac{\Delta x_3}{\Delta} = \frac{-42}{-42} = 1.$$

3. Matritsaviy usul: Bu usulda sistemaning matritsaviy ko‘rinishida yozilgan ifodasidan foydalaniladi.Bunda r(A)=n shartdan sistemaning n-tartibli A kvadrat matritsasi maxsusmas ekanligi kelib chiqadi, chunki matritsa rangi ta‘rifiga asosan

$\Delta = |A| \neq 0$ bo‘ladi. Bu holda A matritsaga teskari matritsa A^{-1} mavjud va bu matritsaviy tenglamaning ikkala tomonini unga chap tomondan ko‘paytirish mumkin. Natijada,teskari matritsa ta‘rifi va birlik matritsa xossasidan foydalanib, ushbu formulani hosil etamiz.

$$A \cdot x = B \to A^{-1}A \cdot x = A^{-1} \cdot B \to E \cdot x = A^{-1} \cdot B \to x = A^{-1} \cdot B.$$

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \cdots + a_{2n}x_n = b_2 \\ \ldots \\ a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \cdots + a_{mn}x_n = b_m \end{cases}$$

berilgan bo‘lsin.

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} & \ldots a_{1n} \\ a_{21} & a_{22} & a_{23} & \ldots a_{2n} \\ & & \ldots & \\ a_{m1} & a_{m2} & a_{m3} & \ldots a_{mn} \end{pmatrix}$$

$$x = \begin{pmatrix} x_1 \\ x_2 \\ \ldots \\ x_m \end{pmatrix}; \qquad B = \begin{pmatrix} b_1 \\ b_2 \\ \ldots \\ b_m \end{pmatrix}; \qquad A^{-1} = \frac{1}{\Delta} \cdot \begin{pmatrix} A_{11} & A_{21} & A_{31} & \ldots A_{m1} \\ A_{12} & A_{22} & A_{32} & \ldots A_{m2} \\ & & \ldots & \\ A_{1n} & A_{2n} & A_{3n} & \ldots A_{mn} \end{pmatrix}$$

$x = A^{-1} \cdot B$ formula orqali hisoblanadi.

5-misol:

$$\begin{cases} x + 2y + 3z = 2 \\ 2x + 3y - 4z = -5 \\ 3x + y + z = 3 \end{cases}$$

$$\Delta = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & -4 \\ 3 & 1 & 1 \end{bmatrix} = 3 + 6 - 24 - 27 - 4 + 4 = -42$$

$$A_{11} = \begin{bmatrix} 3 & -4 \\ 1 & 1 \end{bmatrix} = 7, \quad A_{12} = -\begin{bmatrix} 2 & -4 \\ 3 & 1 \end{bmatrix} = -14, \quad A_{13} = \begin{bmatrix} 2 & 3 \\ 3 & 1 \end{bmatrix} = -7,$$

$$A_{21} = -\begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix} = 1, \quad A_{22} = \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix} = -8, \quad A_{23} = -\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix} = 5,$$

$$A_{31} = \begin{bmatrix} 2 & 3 \\ 3 & -4 \end{bmatrix} = -17, \quad A_{32} = -\begin{bmatrix} 1 & 3 \\ 2 & -4 \end{bmatrix} = 10, \quad A_{33} = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} = -1$$

$$A^{-1} = \frac{1}{-42} \cdot \begin{pmatrix} 7 & 1 & -17 \\ -14 & -8 & 10 \\ -7 & 5 & -1 \end{pmatrix}$$

$$x = A^{-1} \cdot B = \frac{1}{-42} \cdot \begin{pmatrix} 7 & 1 & -17 \\ -14 & -8 & 10 \\ -7 & 5 & -1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -5 \\ 3 \end{pmatrix} =$$

$$= \begin{pmatrix} \frac{-14}{42} + \frac{5}{42} + \frac{51}{42} \\ \frac{28}{42} - \frac{40}{42} - \frac{30}{42} \\ \frac{14}{42} + \frac{25}{42} + \frac{3}{42} \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$$

Xulosa qilib shuni aytish mumkinki, har bir usul o‘ziga xos afzalliklarga ega bo‘lib, ularni o‘z o‘rnida qo‘llash hisoblash ishlarini sezilarli darajada osonlashtiradi. Maqolada keltirilgan nazariy tushunchalar va amaliy misollar oliy ta’lim muassasalari talabalari uchun chiziqli algebra fanini o‘zlashtirishda hamda amaliy muammolarni matematik usullar bilan hal qilishda metodik manba bo‘lib xizmat qiladi.

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