Oriental Renaissance: Innovative, educational, natural and social sciences
SJIF 2024 = 7.404 / ASI Factor = 1.7
(E)ISSN: 2181-1784
4 (3), March., 2024
www.oriens.uz

EXTENSION AND APPLICATION OF NEWTON'S METHOD IN NONLINEAR OSCILLATION THEORY

Mamatova Mahliyo Adhamovna
Teacher at the Department of Physics, Fergana State University,
Mamirjonova Gulnoza Shokirjon qizi
Master, Fergana State University

We all know that because the nonlinear oscillation is very important in theory and application, it is a main subject studied by mathematicians and mechanicians. The analytic method in the study of nonlinear oscillation is most important.

Because derivative systems of quasi-linear systems are the simplest linear ordinary differential equations with constant coefficients and the small parameters appear in the equations, the homotopy invariants between the original systems and derivative system was quite easily set up. Therefore, the theory and the method for studying the periodic solution to quasi-linear systems have already been obtained, such as asymptotic method, small parameter method and method of multiple scales $[2;-4]$. However, the study of general strong nonlinear systems is very difficult. The main difficulty is that in the general case the concept of the derivative systems is not clear. Even if the small parameters exist in the control equations, the derivative systems also show strong nonlinearity. Therefore, it is very difficult to find out the integration of the derivative systems. In spite of this, scholars of many countries have done a lot in the strong nonlinear oscillation. So far, the methods in the study of quasi-linear systems have been extended to the study of strong nonlinear oscillation, references [5;-6] apply asymptotic method to investigate quasi-conservative system and in reference [7] Burton discussed the similar problem by using time transfomation method. But at present it is evident to lack the unanimous effective analytic method for studying strong nonlinear nonautonomous systems, especially the case of large amplitude excitation. Although in some cases we can obtain the approximate periodic solutions of strong nonlinear systems by using the. approximate methods as direct variational method and Galerkin method and so on, to raise the calculation accuracy makes unknown numbers increase, sothat solving the transcendental equations is very difficult, and at the same time it is not probable to obtain the asymptotic analytic solutions with explicit occurrence. Therefore, the application of these direct approximate methods is restricted. Besides, Liapnov system method developed by I.G. Malkin is only suitable to small amplitude oscillation.

On the other hand, we note that Newton's method used long ago to solve the algebraic and transcendental equations has the characters of making the nonlinear problems transformed into the linear problems, alternating programs being simpler with fast speed of convergence.

The eminent former Soviet scholar L.V.Contolovich first successfully applied Newton's method to study abstract operators in the functional, space, and at the same time he suggested simplified Newton's method and studied the existence of the periodic solution for the second order nonlinear system about some class. But his proof can not give the analytic method to calculate the periodic solutions.

In this paper we give up the classcial nonlinear method, that is, we use the method to find out the periodic solution of the derivative system. We directly establish the approximate concept based upon Newton's method 'and prove that general second order nonlinear systems may have the periodic solution, whose result is wider than L. V. Contolovich's, and at the same time we'can give an iteration method to calculate asymptotic analytic solutions of strong or weak nonlinear systems. Finally we'll further prove by some examples that the method we use in this paper is feasible.

We consider

$$\ddot{x}(t) = \phi(t) \cdot x(t) + \psi(t)\dot{x}(t) + f(t) \qquad (1)$$

and assume that $\phi(t)$ $\Psi(t)$ and $f(t)$ are all continuous periodic functions of periodic $2\pi/\omega$ so, they can be expanded into Fourier series of the complex form as follows:

$$\phi(t) = \sum_{n=-\infty}^{\infty} \xi_n \cdot exp[in\omega t] \qquad (2)$$

$$\psi(t) = \sum_{n=-\infty}^{\infty} \zeta_n \cdot exp[in\omega t] \qquad (3)$$

$$f(t) = \sum_{n=-\infty}^{\infty} a_n \cdot exp[in\omega t] \qquad (4)$$

We let

$$x(t) = \sum_{n=-\infty}^{\infty} a_n \cdot exp[in\omega t] \qquad (5)$$

Putting (2 -4) and (5) into (1) and equating the coefficients of the same harmonics on both sides yields

$$\begin{cases} a_0 + \sum_{r=-\infty}^{\infty}(\xi_{-r} + \delta(r) + ir\omega\zeta_{-r})a_r = -a_0 \\ a_n + \frac{1}{n^2\omega^2} \cdot \sum_{r=-\infty}^{\infty}(\xi_{n-r} + \delta(r) + ir\omega\zeta_{n-r})a_r = -\frac{a_n}{n^2\omega^2} \end{cases} \qquad \text{(6a) and (6b)}$$

$$(n = \pm 1, \quad \pm 2, \ \dots.)$$

Where

$$\delta(r) = \begin{cases} -1, & r = 0 \\ 0, & r \neq 0 \end{cases} \qquad (7)$$

$$M = \{\sum_{n=0}^{\infty} |\xi_n|^2\}^{\frac{1}{2}} \qquad (8)$$

$$Q = \{\sum_{n=0}^{\infty} |\zeta_n|^2\}^{\frac{1}{2}} \qquad (9)$$

$$R = \{\sum_{n=0}^{\infty} |a_n|^2\}^{\frac{1}{2}} \qquad (10)$$

In addition, we also assume $\psi(t)$ is continuous, so on the strength ofdifferentiable theorem term by term to Fourier series in reference [9] we have

$$N = \{\sum_{n=\infty}^{\infty} n^2 |\zeta_n|^2\}^{\frac{1}{2}} < +\infty \qquad (11)$$

And let the integer

$$r^*(n) = \min(\{r \downarrow r^2 < 2(n-r)^2\} \qquad (12)$$

Then

$$r^*(n) = 4n \qquad (13)$$

It clearly visible that when $\forall\, r > r^*(n)$ we $r^2 < 2(n-r)^2$ So considering (8 - 9),

(11 - 12) and (13), we have

$$\sum_{|n|=1}^{\infty} \sum_{r=-\infty}^{\infty} \left|\frac{\xi_{n-r} + ir\omega\zeta_{n-r}}{n^4}\right|^2 \leq \sum_{|n|=1}^{\infty} \sum_{r=-\infty}^{\infty} \left|\frac{|\xi_{n-r}|^2 + r^2\omega^2|\zeta_{n-r}|^2}{n^4}\right|^2$$

$$\leq \sum_{|n|=1}^{\infty} \sum_{r=-\infty}^{\infty} \frac{|\xi_{n-r}|^2}{n^4}$$

$$+ \omega^2 \sum_{|n|=1}^{\infty} \sum_{|r|<r^*(n)} \frac{[r^*(n)]^2 \, |\xi_{n-r}|^2}{n^4}$$

$$+ 2\omega^2 \sum_{|n|=1}^{\infty} \sum_{|r|>r^*(n)} \frac{(n-r)^2 \, |\xi_{n-r}|^2}{n^4}$$

$$\leq 2(M^2 + 2\omega^2 N^2) \sum_{n=1}^{\infty} \frac{1}{n^4} + 32\omega^2 Q^2 \sum_{n=1}^{\infty} \frac{1}{n^2} < +\infty$$

Therefore

$$I_1 = \sum_{r=-\infty}^{\infty} |\xi_{-r} + \delta(r) + ir\omega\zeta_{-r}|^2 + \sum_{|n|=1}^{\infty} \sum_{r=-\infty}^{\infty} \frac{|\xi_{-r} + \delta(r) + ir\omega\zeta_{-r}|^2}{n^4} < +\infty \qquad (14)$$

$$\sum_{|n|=1}^{\infty} \frac{|a_n|^2}{n^2} < 2R^2 < +\infty \qquad (15)$$

So

$$I_2 = |a_0|^2 + \sum_{|n|=1}^{\infty} \frac{|a_n|^2}{n^2\omega^2} < +\infty \qquad (16)$$

Let

$$\eta_1 = \left\{ \sum_{|n|>m+1}^{\infty} \sum_{r=-\infty}^{\infty} \left| \frac{\xi_{n-r} + ir\omega\zeta_{n-r}}{n^2\omega^2} \right| \right\}^{\frac{1}{2}} \quad (17)$$

It is clearly visible that

$$\lim_{m\to\infty} \eta_1 = 0 \quad (18)$$

And let

$$\eta_2 = \left\{ \sum_{|n|>m+1}^{\infty} \frac{|a_n|^2}{n^2\omega^2} \Big/ \sum_{|n|=1}^{\infty} \frac{|a_n|^2}{n^2\omega^2} \right\}^{\frac{1}{2}} \quad (19)$$

It is clearly visible that

$$\lim_{m\to\infty} \eta_2 = 0 \quad (20)$$

We can change the form of equations (6) into still simpler one as follows

$$Ka = (I + H)a = \alpha \quad (21)$$

The equations

$$K'a' = (I' + H')a' = \alpha' \quad (22)$$

Are called the cut equations of (6), which can take the place of (6) approximately, where I' is $(2m+1)(2m+1)$ order unit matrix, $a' = [a_n]^T_{n=0,\pm1,\pm2\ldots\pm m}$; H' is $(2m+1)(2m+1)$ order matrix located at the upper left side in H. The operator ϕ represents the following transformation

$$x' = \phi x$$

Where $= [x_f]^T_{i=0,\pm1,\pm2,\ldots.}$. It is clearly visible that

$$\|\phi K\| \le 1 + \|\phi H\| \le 1 + \sum_{|n|=1}^{\infty} \sum_{r=-\infty}^{\infty} \left| \frac{\xi_{n-r} + ir\omega\zeta_{n-r}}{n^4} \right|^2 +$$

$$+ \sum_{r=-\infty}^{\infty} |\xi_{-r} + \delta(r) + ir\omega\zeta_{-r}|^2 < +\infty \quad (23)$$

And

$$\|K'^{-1}\| = \max_{f=0,\pm1,\pm2\ldots\pm m} \frac{1}{1 + \sqrt{|\lambda_f|}} \quad (24)$$

Where $\lambda_f (j = 0, \pm1, \ldots, \pm m)$ are the eigenvalues of the matrix, $H'^* \cdot H'$. H'^* is Hermite adjoint matrix of H'. So from Ref. [8] we know that if the following inequalite is satisfied

$$T = \eta_1 \|K'^{-1}\| \cdot \|\phi K\| < 1 \qquad (25)$$

Then when, $m \to \infty$, the solution of (22) must converge to the solution of (21). From formula (24), it is clear that norms $\|K'^{-1}\|$ for all m, which are arbitrary integers, must have the upper boundary. By this and considering (23) and (28), we know when m is sufficiently large integer, formula (25) can always be established. Therefore if the cut equations (22) have the solution, then when, $m \to \infty$, the solution of (22) must converge to the solution of (21).

Now we will prove that periodic solution of (1) obtained by means of the above method is absolute uniform convergence. For the sake of our purpose, substituting (5) into (1) and considering (8 – 9) and (10), we obtain

$$n^2\omega^2|a_n| \le \mu(1+|n|)|a_n| + R \qquad (n = \pm 1, \pm 2, \dots) \qquad (26)$$

where

$$\mu = max\{M, \omega Q\} \qquad (27)$$

When $|a_n|$ is sufficiently large, so that $|a_n| > s$, where the integer s satisfies the following inequality

$$s > \frac{1}{2}\left\{\frac{\mu}{\omega^2} + \sqrt{\frac{\mu^2}{\omega^4} + \frac{4\mu}{\omega^2}}\right\} \qquad (28)$$

So we have

$$|a_n| \le \frac{R}{n^2\omega^2 - \mu(1+|n|)} \qquad (n = \pm 1, \pm 2, \dots) \qquad (29)$$

Therefore, from (5) and (29) we obtain

$$|x(t)| \le \sum_{|n|=0}^{\infty} |a_n| \le \sum_{|n|=0}^{s} |a_n| + R \sum_{|n|>s}^{\infty} \frac{1}{n^2\omega^2 - \mu(1+|n|)} \qquad (30)$$

It is clearly visible that the right-hand side of (30) is a bounded positive value, from which we can arrive at the following conclusion.

REFERENCES

1. Mamatova, M. A., Yavkachovich, R. R., Dilshodbek, M., & Forrukh, K. (2022). Relation between the concentration of nonequilibrium electrons and holes in long semiconductor diodes. *European science review*, (5-6), 29-32.

2. Расулов, В. Р., Расулов, Р. Я., Маматова, М. А., & Исомаддинова, У. М. (2022). К ТЕОРИИ ЭЛЕКТРОННЫХ СОСТОЯНИЙ В МНОГОСЛОЙНОЙ ПОЛУПРОВОДНИКОВОЙ СТРУКТУРЕ. КВАЗИКЛАССИЧЕСКОЕ ПРИБЛИЖЕНИЕ. *Universum: технические науки*, (10-5 (103)), 24-31.

3. Rustamovich, R. V., Yavkachovich, R. R., Adhamovna, M. M., Qizi, K. M. N., & Dovlatboyevich, M. D. (2022). VOLT-AMPERE CHARACTERISTICS OF A THREE-LAYER SEMICONDUCTOR DIODE OF DOUBLE INJECTION. *European science review*, (5-6), 37-41.
4. Rasulov, V. R., Rasulov, R. Y., Mamatova, M. A., & Eshboltaev, I. M. (2022). THEORETICAL INVESTIGATION OF ENERGY STATES IN A MULTILAYER SEMICONDUCTOR STRUCTURE IN THE QUASICLASSICAL APPROXIMATION. *Galaxy International Interdisciplinary Research Journal*, *10*(12), 96-104.
5. Rasulov, V. R., Rasulov, R. Y., Mamatova, M. A., & Qosimov, F. (2022, December). Semiclassical theory of electronic states in multilayer semiconductors. Part 1. In *Journal of Physics: Conference Series* (Vol. 2388, No. 1, p. 012156). IOP Publishing.
6. Маматова, М. А., Исомаддинова, У. М., Кодиров, Н. У. О., & Обидова, М. И. (2022, December). КВАЗИСТАЦИОНАРНЫЕ ЭНЕРГЕТИЧЕСКИЕ СОСТОЯНИЯ В СФЕРИЧЕСКОЙ ПОЛУПРОВОДНИКОВОЙ ПОТЕНЦИАЛЬНОЙ ЯМЕ. In *The 12 th International scientific and practical conference "Eurasian scientific discussions"(December 18-20, 2022) Barca Academy Publishing, Barcelona, Spain. 2022. 542 p.* (p. 226).
7. Rasulov, V. R., Rasulov, R. Y., Mamatova, M. A., & Eshboltaev, I. M. (2022). THEORETICAL INVESTIGATION OF ENERGY STATES IN A MULTILAYER SEMICONDUCTOR STRUCTURE IN THE QUASICLASSICAL APPROXIMATION. *Galaxy International Interdisciplinary Research Journal*, *10*(12), 96-104.
8. Rasulov, V. R., Rasulov, R. Y., Mamatova, M. A., & Gofurov, S. Z. U. (2022). GENERALIZED MODEL FOR THE ENERGY SPECTRUM OF ELECTRONS IN TUNNEL-COUPLED SEMICONDUCTOR QUANTUM WELLS. *EPRA International Journal of Multidisciplinary Research (IJMR)*, *8*(12), 1-5.
9. Rasulov, V. R., Rasulov, R. Y., Mamatova, M. A., & Qosimov, F. (2022, December). Semiclassical theory of electronic states in multilayer semiconductors. Part 2. In *Journal of Physics: Conference Series* (Vol. 2388, No. 1, p. 012158). IOP Publishing.
10. Rasulov, V. R., Rasulov, R. Y., Mamatova, M. A., & Eshboltaev, I. M. (2022). THEORETICAL INVESTIGATION OF ENERGY STATES IN A MULTILAYER SEMICONDUCTOR STRUCTURE IN THE QUASICLASSICAL APPROXIMATION. *Galaxy International Interdisciplinary Research Journal*, *10*(12), 96-104.

11. Rasulov, V. R., Rasulov, R. Y., Mamatova, M. A., & Gofurov, S. Z. U. (2022). GENERALIZED MODEL FOR THE ENERGY SPECTRUM OF ELECTRONS IN TUNNEL-COUPLED SEMICONDUCTOR QUANTUM WELLS. *EPRA International Journal of Multidisciplinary Research (IJMR)*, *8*(12), 1-5.
12. Расулов, Р. Я., Маматова, М. А., Хасанович, Р., & Муминов, И. (2014). О кинетическом уравнении для дырок в размерно-квантованной яме полупроводника вырожденной валентной зоной. *Austrian Journal of Technical and Natural Sciences*, (9-10), 150-153.
13. Rasulov, R. Y., Karimov, I. N., Mamatova, M., Omonova, H., & Sultanov, R. R. European Science Review, Issue 1-2-1/2019.
14. Маматова, М. А., & Расулов, Р. Я. (2019). К ТЕОРИИ ВОЛЬТ-АМПЕРНОЙ ХАРАКТЕРИСТИКИ ТРЕХСЛОЙНОЙ СТРУКТУРЫ ПОЛУПРОВОДНИКОВ В ДИОДНОМ ВКЛЮЧЕНИИ. In *WORLD SCIENCE: PROBLEMS AND INNOVATIONS* (pp. 13-17).
15. Rasulov, V. R., Rasulov, R. Y., & Adhamovna, M. M. (2022). ELECTRONIC PROPERTIES OF A SEMICONDUCTOR TWO-BARRIER STRUCTURE. *EPRA International Journal of Multidisciplinary Research (IJMR)*, *8*(5), 58-62.